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IMPLEMENTATION OF THE SOCIALLY OPTIMAL OUTCOME*

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We show that a welfare maximizing planner in a Cournot oligopoly can easily implement the socially optimal outcome by offering the firms a per unit subsidy in return for upfront fees. The planner announces a subsidy and auctions it off to a limited number of firms. It is shown that if at least one firm is excluded and not subsidized, the socially optimal outcome can be achieved while the planner runs no deficit. The planner does not impose any regulation on the firms. They accept his offer willingly and voluntarily. Yet, every firm makes zero net profit and consumers extract the entire surplus.

1 Introduction

A central planner in a free society would rather design a mechanism that implements a certain outcome than impose this outcome directly. A proper mechanism provides the agents with the right incentive to choose actions (individually and voluntarily) that would dictate the desirable outcome.

This paper deals with a central planner (government) who acts in a Cournot oligopoly industry. The planner's objective is to maximize social welfare. The purpose of this paper is to show the existence of a simple mechanism offered by the planner such that (i) every firm is free to choose whether or not to take part in the game generated by the mechanism (individual rationality),[1] (ii) the socially optimal outcome is implemented, (iii) the planner keeps a balanced budget, and (iv) the planner takes no part in the production and the selling process.[2] To achieve this goal, the planner offers a subsidy to a number of firms for each unit of output they produce. The per unit subsidy and the number of subsidized firms are the strategic choices of

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[1] Individual rationality eliminates a price cap imposed by the planner.
[2] This rules out an acquisition of one firm by the planner together with a commitment to produce the competitive output.

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the planner. Once they are announced, the firms that wish to be subsidized bid for the planner's offer, and the winning bids are collected upfront.

We emphasize that the planner does not impose any regulation on the firms, nor does he buy or sell their products. He plays no role other than offering the firms a per unit subsidy, in return for upfront fees, determined by the firms themselves in a first-price auction. It is shown that the planner can choose the number of subsidized firms and the subsidy level, in such a way that the resulting Cournot outcome is socially optimal. Furthermore, the planner breaks even as he collects upfront fees equal to future subsidies. Every firm is worse off as it ends up with zero net profit; namely, firms are voluntarily willing to pay their entire profit to the planner who in turn offers them sufficiently high subsidies, which reduces the Cournot market price to the marginal cost of production. The non-subsidized firms exit the market and the consumers extract the entire surplus. An interesting observation is that the socially optimal outcome cannot be achieved without exclusion. The planner must exclude at least one firm and cannot subsidize all the firms in the industry.

A similar phenomenon is observed by Lahiri and Ono (1988)[3] and by Tandon (1984), who suggest that an industry with fewer firms may be socially preferable. In the first paper, it is shown that in a Cournot oligopoly with uneven technologies, improvement in production efficiency may exceed diseconomies of the changes in market structure. The second paper argued that free entry to an industry engaged in R&D competition would lead to excessive entry in the sense that an industry with fewer firms would be socially preferable.

In a related paper, Bar Niv and Zang (1999) have shown that the government can costlessly increase efficiency in a (natural) monopoly market with potential entrants by subsidizing the entry cost of entrants. As a result, the incumbent monopolist drops his price to the marginal cost; hence no entry occurs and no subsidy is actually paid. This result can be achieved only with some government intervention; namely, after entry occurs, the incumbent is forced to choose (and to commit to) a certain output level before the entrants make their own output choices.

Finally, White (1996) has shown that optimal linear subsidization of all the firms of a Cournot oligopoly industry (with quadratic cost functions) can improve total welfare. The planner, however, is running a deficit and the socially optimal outcome is not achieved.

2 The Model

Consider a Cournot oligopoly industry with $n \geq 2$ firms producing a homogeneous good with identical constant returns to scale technology. This

[3]This paper was inspired by Katz and Shapiro (1985).

technology is represented by the cost function $f(q) = cq$, where q is the quantity produced and c is the marginal cost of production. The aggregate demand is $Q(p)$, where p is the Cournot market price, and $\eta(p)$ is the price elasticity of demand, namely, $\eta(p) = -pQ'/Q$. It is assumed that $\eta(p)$ is a non-decreasing function of p.

Consider an entity G (government) that credibly commits to pay a per unit subsidy, s, to k firms, $0 \le k \le n$, in return for an upfront fee, which is determined by a first-price auction; namely, G announces a subsidy s and auctions off k contracts. Bids must be non-negative. A contract between G and a firm is a commitment of G to pay this firm a predetermined price s for every unit it produces (hence decreasing its marginal cost from c to $c - s$). In return, G collects the bids of the k highest bidding firms.[4] In equilibrium, all the winning firms submit the same bid, which is the difference between the profit of a subsidized firm and the profit of a non-subsidized firm, given that there are k subsidized firms. Furthermore, if $1 \le k \le n - 1$, the number of firms that submit the winning bid is at least $k + 1$; otherwise, every winning firm is better off by slightly reducing its bid. Note that if a winning firm deviates and reduces its bid, the number of subsidized firms will continue to be k. This is not the case when $k = n$—a deviating firm will face $n - 1$ subsidized firms.

Let T be the set of the firms that win the auction, $|T| = k$. The operating profits of the firms are

$$\Pi_i = \begin{cases} q_i(p-c+s) & i \in T \\ q_i(p-c) & i \notin T \end{cases}$$

where q_i is the Cournot quantity produced by firm i. Taking the first-order conditions with respect to q_i, it can be verified that

$$1-\frac{1}{n\eta(p)}=\frac{c-(k/n)s}{p} \qquad \text{for } k \le \frac{c}{s\eta(c)} \tag{1a}$$

and

$$1-\frac{1}{k\eta(p)}=\frac{c-s}{p} \qquad \text{for } k \ge \frac{c}{s\eta(c)} \tag{1b}$$

Since $\eta(p)$ is non-decreasing in p, equations (1a) and (1b) uniquely determine, for any k, $0 \le k \le n$, an equilibrium price $p = p(k)$.

Lemma 1: The Cournot price $p(k)$ is decreasing in k for $1 \le k \le n$.

[4]If (a) the minimum winning bid is $\underline{b}$, (b) m bidders bid above $\underline{b}$ and (c) l bidders bid $\underline{b}$ such that $m + l > k$, then out of the l lower bidders, $k - m$ bidders are chosen at random. The result does not change whether the auction is discriminatory, namely, the winners pay their own bids, or if each one of them pays just $\underline{b}$.

Proof: The proof is obtained by differentiating both sides of (1a) and (1b) with respect to p while using the assumption that $\partial\eta/\partial p \geq 0$. For a formal proof, see Lemma 1 of Kamien *et al*. (1992). ■

Note that if we substitute $p = c$ in (1a) (or in (1b)), we have $k = c/s\eta(c)$. By Lemma 1, $p(k)$ is decreasing in k and thus $k = c/s\eta(c)$ is the unique solution to the equation $p(k) = c$. Hence, if s is the per unit subsidy, then

$$k = \frac{c}{s\eta(c)} \Leftrightarrow p(k) = c \tag{2a}$$

and

$$k > \frac{c}{s\eta(c)} \Leftrightarrow p(k) < c \tag{2b}$$

The Cournot quantities for $k \leq c/s\eta(c)$ are

$$q_i = \begin{cases} \dfrac{Q(p)[c - s + s(n-k)\eta(p)]}{nc - ks} & i \in T \\ \dfrac{Q(p)[c - sk\eta(p)]}{nc - ks} & i \notin T \end{cases} \tag{3}$$

where $p = p(k)$. For $k \geq c/s\eta(c)$

$$q_i = \begin{cases} \dfrac{Q(p)}{k} & i \in T \\ 0 & i \notin T \end{cases} \tag{4}$$

The Cournot operating profits of the firms for $k \leq c/s\eta(c)$ are

$$\Pi_i(k) = \frac{q_i^2}{\eta(p)} \frac{p}{Q(p)} \qquad i = 1, \ldots, n \tag{5}$$

By (4)

$$k \geq \frac{c}{s\eta(c)} \Rightarrow \Pi_j(k) = 0 \qquad \text{for } j \notin T \tag{6}$$

Let $1 \leq k \leq n - 1$ and let $s(k) = c/k\eta(c)$. Then, by (2a) $p(k) = c$. By (6), if a winning firm deviates and reduces its bid, it will make zero profit.[5] Hence, when $1 \leq k \leq n - 1$ and $s = s(k)$, every firm is willing to pay G its entire profit for a contract. This implies that every firm, whether it is subsidized or not, ends up making zero profit and the total social surplus is just the sum of G's net income and the consumer surplus. Let $1 \leq k \leq n - 1$ and let $\Pi_G(k, s(k))$ be G's net income under $(k, s(k))$. Since $p(k) = c$, we have

[5]Recall that if a winning firm deviates and reduces its bid, the number of subsidized firms will continue to be k and the Cournot price will continue to be c.

$$\Pi_{\mathrm{G}}(k, s(k)) = \sum_{i \in T} [p(k) - c + s]q_i - \sum_{i \in T} sq_i = 0$$

Namely, G just breaks even and the total social welfare is the consumer surplus at $p = c$. This is the socially optimal outcome. We summarize the result above in the following proposition.

Proposition 1: An entity G can implement the socially optimal outcome without running a deficit by subsidizing at most $n - 1$ firms. The optimal subsidy is $s(k) = c/k\eta(c)$, where k is the number of subsidized firms and $1 \le k \le n - 1$. This subsidy reduces the Cournot price to the marginal cost of production, c, and non-subsidized firms exit the market. Every firm makes zero profit, G just breaks even and consumers extract all the surplus.

Remark 1. There are other simple mechanisms that induce the same outcome, where the socially optimal outcome is achieved costlessly for the government and voluntarily for the firms. For example, the government offers an exclusive contract to pay a single firm a certain fee in return for its commitment to produce and sell at least $Q(c)$ units in the market. The government procures this contract through an auction where the fee is the lowest bid.[6] Indeed, in equilibrium, at least two firms bid zero (knowing that they will earn zero if they do not win the contract), the winning firm produces $Q(c)$ units and the government pays zero. However, the equilibrium here uses (weakly) dominated strategies. Bidding zero is a dominated strategy for every firm. If a firm i wins the contract it will earn at most zero, and if some other firm produces a positive amount then i will incur a loss. If j, $j \ne i$, wins the contract, then i will obtain zero if it exits the market and will incur a loss otherwise.

In contrast, bidding the entire profit is not a dominated strategy in our mechanism whenever $k \ge 2$. A winning firm i obtains, in equilibrium, zero net profit but would obtain a positive net profit if another winning firm produces below its Cournot profit level, driving the price above the marginal cost level.

Remark 2. The assumption that the firms are symmetric is not crucial. Suppose that the marginal costs of the firms are $c_1 \le c_2 \le \ldots \le c_n$. Let

$$I = \{i | c_i = c_1\}$$

Our mechanism can be modified in a straightforward way to deal with the case where $|I| \ge 2$. In this case the government offers the firms in I an exclusive contract to pay the highest bidder in I the per unit subsidy $s = c_1/\eta(c_1)$. The equilibrium outcome is the same as in Proposition 1 where c is replaced by c_1.

[6]We thank an anonymous referee who provided us with this example.

If $I = \{1\}$, to obtain the same result we introduce the following sequential mechanism. The government makes a 'take it or leave it' offer to the firms in a sequence where the most efficient firm (firm 1) is approached first. It is offered a per unit subsidy $s_1 = c_1/\eta(c_1)$ in return for its Cournot profit. If the offer is accepted, firm 1 is the subsidized firm. If firm 1 rejects the offer, then firm 2 is approached and it is offered the per unit subsidy $s_2 = c_2 - c_1 + c_1/\eta(c_1)$ in return for its Cournot profit, and so on; firm k, $1 \leq k \leq n - 1$, is offered the per unit subsidy $s_k = c_k - c_1 + c_1/\eta(c_1)$ in return for its Cournot profit. Finally, if all the first $n - 1$ firms reject their offers, firm n is approached and is offered the per unit subsidy $s_n = c_n - c_1 + c_1/\eta(c_1)$, but this time it is offered, in addition, a lump-sum amount δ satisfying $\delta > \Pi_n^{\mathrm{O}} - \Pi_n^{\mathrm{M}}$, where Π_n^{O} is the Cournot oligopoly profit of firm n, in the case where no firm is subsidized, and Π_n^{M} is monopoly profit of a firm operating with a constant marginal cost of $c_1[1 - 1/\eta(c_1)]$.

Note that if a firm k, $1 \leq k \leq n$, accepts the offer, then the Cournot price will be c_1 and all the other firms exit the market. As for firm n, if it rejects the offer, then no firm is subsidized and firm n will earn Π_n^{O}. If it accepts the offer, it will obtain Π_n^{M}. Since $\delta > \Pi_n^{\mathrm{O}} - \Pi_n^{\mathrm{M}}$, firm n will accept the offer. Thus, firm $n - 1$ knows that if it rejects the government's offer it will be driven out by firm n, and hence it will accept the offer. Using the backward induction argument, firm 1 will accept the offer and will become the only subsidized firm. The other firms will exit the market, the government will break even and the market price will be c_1, the marginal cost of the most efficient firm.

Remark 3. Our linear subsidy scheme fits only the constant marginal costs case. In a sequel paper, Sengupta and Tauman (2004) consider a Cournot oligopoly market of firms processing increasing returns to scale technologies. They show the existence of a non-linear subsidy scheme, which depends on both the output level of the subsidized firm and the market price such that the government breaks even and the subsidized firm obtains zero net profit and charges a price equal to its average cost. Every other firm exits the market, consumers are better off and the total welfare improves. The socially optimal outcome, where price equals marginal cost, cannot be achieved.

We have shown that the socially optimal outcome can be achieved with the subsidy $s(k) = c/k\eta(c)$ and with k subsidized firms, while running no deficit. Our next goal is to show that this is impossible if $k = n$, namely, when G subsidizes all the firms.

Consider the case $k = n$. The willingness to pay of every firm is now zero since G is committed to subsidize every firm. To avoid this outcome, G may set a floor price which is the difference between the profit of a subsidized firm and the profit of a non-subsidized firm, given that all other firms are subsidized. Clearly, the winning bids are equal to this floor price and they are given by

$$b_i = \Pi_i(n) - \tilde{\Pi}_j$$

where $\tilde{\Pi}_j$ is the Cournot profit of a non-subsidized firm when it competes with $n - 1$ subsidized firms. To achieve the socially optimal outcome, $p(n) = c$ must hold. This is obtained iff $s = c/n\eta(c) = s(n)$. By Lemma 1, $p(n - 1) > c$ and thus $\tilde{\Pi}_j > 0$. But now

$$\begin{aligned} \Pi_G(n, s(n)) &= \sum_{i=1}^{n} q_i[p(n) - c + s(n)] - n\tilde{\Pi}_j - \sum_{i=1}^{n} s(n)q_i \\ &= -n\tilde{\Pi}_j < 0 \end{aligned} \tag{7}$$

and G runs a deficit.

We summarize the above with the following proposition.

Proposition 2: To implement the socially optimal outcome without running a deficit, G must exclude at least one firm and subsidize at most $n - 1$ firms.

While Proposition 1 asserts that G can implement the socially optimal outcome, without running a deficit, by subsidizing at most $n - 1$ firms, Proposition 2 asserts that exclusion is necessary.

Finally, using the argument above we can show that basically it is necessary to use the auction mechanism to ensure that G does not run a deficit. Consider the case where G puts on sale subsidy contracts for some upfront predetermined price. If G offers the subsidy s and wishes to subsidize exactly k firms, it has to charge

$$\alpha(k) = \Pi_i(k) - \Pi_j(k-1) \qquad i \in T,\ j \notin T$$

per contract. A subsidized firm that regrets buying the contract will compete with $k - 1$ subsidized firms and will obtain $\Pi_j(k - 1)$, $j \notin T$. Hence, the willingness to pay for a contract is $\alpha(k)$. Furthermore, to achieve the socially optimal outcome, $p(k) = c$ must hold and G has to offer the subsidy $s = s(k)$ for the fee $\alpha(k)$ in order to attract exactly k buyers. By Lemma 1, $p(k - 1) > c$ and then $\Pi_j(k - 1) > 0$. Hence, similar to (7)

$$\begin{aligned} \Pi_G(n, s(n)) &= \sum_{i \in T} q_i[p(k) - c + s(k)] - k\Pi_j(k-1) - \sum_{i \in T} s(k)q_i \\ &= -k\Pi_j(k-1) < 0 \end{aligned}$$

and G runs a deficit.

3 Conclusion

It is shown that the exclusion of at least one firm (from being subsidized) is a necessary and sufficient condition for the implementation of the socially optimal outcome without running a deficit. This result is obtained while G is not imposing any regulation on the firms. The only action taken by G is an offer to subsidize a limited number of firms. The firms can either accept the offer or reject it. They voluntarily accept G's offer even though they end up with zero profits and consumers extract the entire surplus.

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