

# Modest advertising signals strength

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## Abstract

*We reexamine the role of prices and advertising expenditures as signals of quality. Consumers are either “fastidious” or “indifferent”. Fastidious individuals value high quality more and low quality less than indifferent individuals. Then a sensible and robust separating equilibrium exists where both types set their full information prices. However, the high-quality firm cuts advertising below the full information level of the low-quality firm, even if the full information advertising expenditures of the high-quality firm are larger than those of the low-quality firm. Consumers respond favorably to advertising cuts and correctly identify quality. Hence, modest advertising may signal high quality.*

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## 1. Introduction

We provide an equilibrium analysis of a model where firms with high-quality products strategically choose to *cut* advertising expenditures, resulting in loss of market volume, to credibly signal their quality. Potential customers who observe advertising expenditures respond positively to these advertising cuts and correctly identify the product type. This result is in contrast to most of the existing literature on signaling games.

The main result of the signaling games literature is that a strong type of player (the one who wishes to reveal his identity) sends a signal which is stronger than the signal he would have sent had there been no doubt as to his identity. Thus, the strong type widens the gap between his signal and the full information signal of the weak type. This relies on the common assumption, known as the “single-crossing condition” (SCC), that strengthening the signal is costlier for the weak type than for the strong type.

This idea was first introduced by Spence (1974) in a setting where “overinvestment” in education serves as a signal of productivity.<sup>1</sup> To name just a few, other models view capital structure as a signal of an entrepreneur’s quality (Ross, 1977), warranties as a signal of product quality (Grossman, 1980), low price as a signal of low production costs (Milgrom and Roberts, 1982), willingness to go to court as a signal of the strength of one’s case (Reinganum and Wilde, 1986), high price as a signal of quality (Riordan (1986), Bagwell and Riordan (1991) and Bagwell (1992)) and low price as a signal of attractive mergers (Saloner, 1987). All of these explicitly or implicitly assume some version of SCC.<sup>2</sup>

Signaling models with *multidimensional* signals include Bagwell and Ramey (1988), who use

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<sup>1</sup>Subsequently, this was further discussed in Cho and Kreps (1987).

<sup>2</sup>Theoretical models which study signaling games that satisfy an appropriate monotonicity condition or SCC include Mailath (1987), Kohlberg (1990), Cho and Sobel (1990) and Stamland (1999).

both price and advertising to signal the cost of production. Bagwell and Ramey (1990) and, in a different context Albaek and Overgaard (1992), use price and advertising expenditures to signal demand intensity. All the above models satisfy the SCC.

Price and advertising expenditures are also used to signal product quality. Milgrom and Roberts' (1986) seminal work uses both price and introductory advertising expenditures as signals of quality. Their model is based on the ideas of Nelson (1970 and 1974), who was the first to argue that repeat purchases of an experience good (where quality is learned only after purchase and use) can create different returns to advertising for different qualities. In Nelson's model, advertising increases the initial sales of all brands equally, but later increases repeat purchases of high-quality brands more. Thus, an initial sale is, *ceteris paribus*, more valuable to firms with a high-quality product, and they are willing to spend more on advertising. This provides the basis for the positive correlation of quality with the net benefits of advertising.

Following Nelson's intuition, Kihlstrom and Riordan (1984) showed that a separating equilibrium exists in which advertising expenditures increase with quality. Their result is obtained either when high-quality production has a low marginal cost, or when the repeat purchase mechanism is sufficiently strong, and in addition marginal costs increase at a very moderate rate as quality is enhanced. These conditions imply the SCC.

Schmalensee (1978) provides a model in which low-quality firms may advertise more than high-quality firms, an observation that is similar to ours. However, to obtain his result Schmalensee requires that consumers follow a non-optimal rule of thumb in making their purchasing decisions, and they do not identify the inverse correlation between advertising and quality. Indeed, Kihlstrom and Riordan (1984, p.428) refer to the possibility that low-quality firms may spend more on advertising as "...inconsistent with rational equilibrium behavior of consumers."

We take a different view by providing a model with fully rational consumers and firms where high-quality firms advertise less than or more than low-quality firms, depending on the parameters of the model. This is in line with the empirical results of Caves and Greene (1996), who failed to find a systematic and significant *positive* correlation between advertising and quality on average in a cross-section of industries.<sup>3</sup>

The existing theoretical literature mostly deals with experience goods and concerns advertising that conveys only *indirect* information about product attributes (i.e., purely dissipative advertising). While our model is similar in spirit, it differs from the above insofar as advertising also conveys *direct* information: a higher advertising level increases the proportion of consumers who become aware of the existence of the product.<sup>4</sup> Thus, demand is directly affected by advertising and not only indirectly through the effect of advertising on consumer perceptions of quality. This is descriptively relevant for the vast majority of new, advertised experience goods. When a car manufacturer, say, advertises a new model on TV, it seems likely that part of the purpose is to make its existence known to potential buyers. In addition, of course, viewers may form their perceptions of quality based on the lavishness and intensity of the advertising campaign.<sup>5</sup>

The model considers the introduction of an experience good produced by a single firm, which can be either of high or low quality. The marginal cost of high quality is *higher* than that of low quality. There are two types of individuals (potential consumers), which we label “fastidious” and “indifferent”. The fastidious individuals place a greater value on the high-quality product

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<sup>3</sup>This, of course, is rather weak evidence. However, it seems fair to suggest that the empirical literature on the relationship between advertising expenditures and product quality is less than conclusive. For a recent review of the evidence, see Kirmani and Rao (2000).

<sup>4</sup>For models where advertising has similar effects, see Bagwell and Ramey (1988) and Albaek and Overgaard (1992).

<sup>5</sup>The fact that signaling theories largely assume that advertising is purely dissipative, while actual advertising may be partly informative causes severe problems for the empirical testing of signaling theories, see e.g. Caves and Greene (1996), Thomas, Shane and Weigelt (1998) and Nichols (1998).

and a lesser value on the low-quality product than do the indifferent individuals.<sup>6</sup>

In the first period, individuals become aware of the product only through the firm's introductory advertising. The proportion of individuals who become aware of the product increases with advertising expenditure, but at a diminishing rate.

Similarly to Milgrom and Roberts (1986), the strategy of the firm consists of two components - the per-period retail price and the level of introductory advertising expenditures - and they can be used to signal the quality of the product.

The phenomenon of "reversed" advertising signals captures that, with incomplete information, a high-quality firm spends *less* on introductory advertising than do low-quality firms, in a setting where the high-quality firm would have spent *more* under full information. To highlight this phenomenon, we impose conditions which guarantee that in every sensible equilibrium both the high-quality firm and the low-quality firm set their full information prices (which are distinct). For this reason, we assume that the proportion of indifferent individuals is significant, and the willingness to pay of the fastidious individuals is not compensatively large.<sup>7</sup> Consequently, the high-quality firm makes sure not to miss out on the purchases of indifferent individuals, and it will be induced to quote their reservation price. This means that the advertising expenditure is in fact the relevant signal of the firm.

We develop sufficient conditions for the existence of a unique "sensible" pure-strategy separating equilibrium with the following properties.<sup>8</sup> The high-quality and the low-quality firm set their full information prices. The full information or *myopic*<sup>9</sup> advertising expenditures of the

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<sup>6</sup>In fact, in the model considered below, "indifferent" individuals are not really indifferent about quality, but they place a small additional value on high quality.

<sup>7</sup>This, of course, includes the case where there are no fastidious individuals at all; a special case to which our formal results will also apply.

<sup>8</sup>By "sensible" we refer to the Intuitive Criterion of Cho and Kreps (1987), which is sufficient given the simplicity of the model.

<sup>9</sup>By myopic we refer to the hypothetical case where every individual who is aware of the existence of the product also knows its quality (see Section 2).

high-quality firm exceed those of the low-quality firm, but, and this is the main result of the paper, in order to signal its type, the high-quality firm substantially *cuts* advertising expenditures to a level *below* even the myopic level of the low-quality firm (hence, the title of the paper).

This is in contrast to the standard result of signaling games, which assume the SCC. The SCC is not an obvious assumption in our model. Strengthening the signal (increasing the advertising expenditures) may sometimes benefit the low-quality firm more than the high-quality firm. This stems from the assumption that the marginal cost of the low-quality firm is smaller than that of the high-quality firm. Thus, when it mimics the high-quality firm, it sets the high-quality price and has a higher mark-up. On the other hand, the high-quality firm generates larger future profits than the low-quality firm due to repeat purchases.

These two opposing effects can either result in the SCC, which in turn yields the standard result in signaling literature, or they can result in violation of the SCC. In the second case, we obtain the signal reversal phenomenon, where the high-quality firm significantly cuts advertising expenditures to a point which makes it unattractive for the low-quality firm to mimic. This result can be obtained, *even* if the mark-up over unit costs is *positively* related to quality.

Intuitively, signal reversal requires two things. *First*, the fraction of indifferent individuals has to be sufficiently large. *Secondly*, the ratio between the mark-ups of the high-quality and the low-quality firm can neither be too large nor too small.

To put this result in perspective, we can relate it to the results of Milgrom and Roberts (1986). When advertising is purely dissipative, as in Milgrom and Roberts (1986), there are basically two ways for the high-quality producer to incur a cost that might reveal his type.<sup>10</sup> He can set a high first-period price and thereby sacrifice volume permanently.<sup>11</sup> Alternatively,

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<sup>10</sup>This relates to the case where unit costs are increasing in quality as in the present paper.

<sup>11</sup>Recall that the formal modelling details of Milgrom and Roberts (1986) are such that only first-period buyers can enter the market in the second period.

he can burn a lot of money in an introductory advertising campaign. The equilibrium mix of the two signaling instruments, of course, depends on the specifics of the given market under scrutiny.<sup>12</sup> In the model of this paper, there is an alternative way for the high-quality producer to incur a cost which might reveal his type. He may choose a modest introductory advertising outlay and thereby sacrifice volume permanently (since he is reaching fewer potential buyers). That this can be an equilibrium phenomenon for a “natural” parameter constellation is the main message of this paper.

Tentatively, this may also explain the mixed empirical evidence of Caves and Green (1996). For a cross-section of industries, the ambiguous relationship between advertising and quality in the pooled sample may simply reflect that in some industries modest advertising is signaling quality, while in others quality is signalled by extensive advertising, and that this is not picked up by any of the explanatory variables. To test our predictions on the relationship between advertising expenditures and quality, control should be made for the key variables emphasized above (the ratio of the mark-ups and the fraction of indifferent individuals).<sup>13</sup> In addition, signal reversal is unlikely to arise if observed advertising is largely dissipative, since low levels of dissipative advertising cannot possibly identify high quality. So, if information can be obtained on the relationship between mark-ups and quality and on the nature of advertising, then the main predictions of this paper are readily testable.

From an empirical perspective, signal reversal in the sense of this paper is more likely to be observed for certain durables such as consumer electronics, household whiteware and computer

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<sup>12</sup>This is often overlooked in empirical investigations, where it is simply stated that theory predicts advertising and quality to be positively related. This, however, is not a general prediction. Milgrom and Roberts (1986) predict that in some cases dissipative advertising will not be used, while in other cases it will be used along-side the price signal. Therefore, failure to find a significant positive relationship between advertising and quality in a *given* sample may simply reflect a preponderance of pure price signaling.

<sup>13</sup>Data on these are not readily available. Mark-ups require data on prices and estimates of unit costs. Sorting of consumers may sometimes be based on simple objective observables (new buyers vs. experienced buyers in a product class such as cars, computer software or hardware, flat screen TVs, play stations, etc.).

hardware than for information goods such as computer software (viz. upgrades), business news and data, etc. The main reason for this is that the unit costs of producing information goods are very low *irrespective* of quality (once first-version sunk cost have been incurred). Hence, the ratio of mark-ups between high and low quality for information goods is typically high, pulling us towards the standard SCC case where high quality is signalled by extensive advertising.<sup>14</sup> In contrast, for many consumer durables, marginal costs are significant and closely tied to quality. If willingness-to-pay only increases moderately with quality, then we would expect to observe high-quality brands being advertised modestly during the introductory phase. Note well what this statement is saying: High-quality products are advertised modestly *compared to* lower-quality products within the same product category. We are *not* saying that the product category *as a whole* (e.g. computer hardware) is advertised modestly. To put this empirical conjecture slightly differently, if gradual and modest quality improvements from one product generation to the next are associated with non-trivial cost increases, then new versions of high-quality products are likely to be advertised modestly.<sup>15</sup>

A slight reinterpretation of our model suggests that restaurants might fit into our framework. High-quality restaurants are very often found in obscure locations off the main street or in small villages, whereas lower-quality outlets such as chain-restaurants and fast-food restaurants are much more conspicuously located at *expensive* main street addresses. So, if location is interpreted as costly and partly informative advertising, then this appears to be a case where quality is signalled by modest advertising outlays.<sup>16</sup>

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<sup>14</sup>Of course, large differences in the willingness-to-pay for high and low qualities also form the basis for versioning and value-based pricing of information goods. Any empirical study will have to account for this as well.

<sup>15</sup>Casual observation suggests that certain luxury cars (such as Rolls Royce and Ferrari) fit this description. New models of these cars seem to be advertised very modestly compared to new models of non-luxury cars, even when account is made for the difference in the market sizes.

<sup>16</sup>Main street addresses are more expensive and a location on main street will (randomly) inform more people about the existence of a particular restaurant. Hence, location formally plays the same role as advertising in our model.

The paper is organized as follows. The next section sets out the model and the full information (or myopic) bench-mark. Section 3 analyses separating equilibria, while Section 4 focuses on signal reversal. Section 5 provides a robust example of signal reversal. In Section 6 we make a few concluding remarks. The Appendix contains most of the proofs.

## 2. The model

A new product is introduced into the market by a monopolist firm. The quality of the product is either “high” ( $H$ ) or “low” ( $L$ ), and this is private information of the firm. Production costs are  $c_H(q)$  and  $c_L(q)$  where  $q$  is the output level. Consumers can discover the quality of the product only if they purchase and use it (i.e., the product launched by the firm is an experience good whose quality is perfectly inferred upon use). We assume the following.

**(A1)** There are two constants  $c_H$  and  $c_L$ , such that  $c_L < c_H$  and  $c_t(q) = c_t q$ ,  $t \in \{H, L\}$ .

The marginal costs are constant and the unit cost of the high-quality product exceeds that of the low-quality product. The linearity assumption is not crucial. It enables us to find explicit and relatively simple conditions for the signal reversal phenomenon to occur. We could however demonstrate our result with quite general cost functions. Actually, if the technology exhibits diseconomies of scale, then the larger the advertising level is, the larger is the output and, hence, the average cost. Consequently, the high-quality firm may find it more advantageous to separate itself from the low-quality firm by cutting advertising rather than by exaggerating advertising. Therefore, it is more challenging (while mathematically simpler) to demonstrate the results with a constant per unit cost function.

There is a continuum of individuals represented by the unit interval  $[0, 1]$ . They are divided into two types: “fastidious” ( $F$ ) and “indifferent” ( $I$ ) in the proportions  $\lambda$  and  $1 - \lambda$ , respectively. Each fastidious individual is willing to pay  $w_H^F$  and  $w_L^F$  per unit of the high-quality and low-

quality goods, respectively. Similarly, each indifferent individual is willing to pay  $w_L^I$  and  $w_H^I$  for the high-quality and low-quality goods, respectively. We make the following assumption.

$$\mathbf{(A2)} \quad 0 \leq w_L^F < w_L^I < w_H^I < w_H^F$$

Essentially this combines two assumptions. A fastidious customer is willing to pay more for a high-quality good, while he is willing to pay less for a low-quality good. Thus, the willingness-to-pay of a fastidious individual is more sensitive to quality than that of an indifferent individual.

The interaction between the firm and the potential customers can be described as follows. In the *first period*, individuals become aware of the product through introductory advertising. The firm chooses the product price,  $p^1$ , and the amount of advertising expenditure,  $x$ . As a result of the advertising, a proportion  $\mu(x)$  of individuals becomes aware of the product. As in most of the literature, it is assumed that  $x$  is known to every individual who becomes aware of the product.<sup>17</sup> We make the following assumption.

$\mathbf{(A3)}$  The function  $\mu(x)$  is differentiable, strictly concave and increasing. In addition,  $\mu(0) = 0$  and  $\mu(x) \rightarrow 1$  as  $x \rightarrow \infty$ .

Thus, the proportion of customers who are exposed to the product increases with the level of advertising expenditures, but at a diminishing rate. With no advertising, this proportion is zero and it increases to one as the advertising expenditures grow indefinitely.

In the *second period*, the price,  $p^2$ , may change, but there is no further advertising. It is assumed that every individual buys either zero or one unit in each period.

To simplify the analysis and focus on the advertising expenditure as the relevant signal, we make two further assumptions.

$$\mathbf{(A4)} \quad w_L^F < c_L < c_H < w_L^I$$

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<sup>17</sup>It is sufficient that exposed individuals have an estimate of  $x$ , based on e.g. the number of times ads observed, their size or broadcasting duration. Most models assume that advertising expenditures are commonly known. One exception is Hertzendorf (1993), who assumes that consumers estimate  $x$  from the number of ads observed.

Thus, the willingness of a fastidious individual to pay for the low-quality product falls short of its cost. This asserts that the low-quality firm will lose by lowering its price from  $w_L^I$  to  $w_L^F$  in order to sell the product to the fastidious individuals as well. In contrast, the willingness of an indifferent individual to pay is sufficient to cover the costs of both qualities. We note that A2 and A4 combine to

$$0 \leq w_L^F < c_L < c_H < w_L^I < w_H^I < w_H^F \quad (1)$$

which we shall refer to repeatedly.

$$\mathbf{(A5)} \quad (1 - \lambda)(w_L^I - c_H) > \lambda(w_H^F - c_H)$$

This condition clearly implies that  $\lambda < 1/2$ . A5 basically asserts that the high-quality firm prefers to set the price at  $w_L^I$ , even if this confuses the consumers into believing that its type is  $L$ , rather than setting the price at  $w_H^F$  and having only the fastidious individuals as its customers. Note that for the special case  $\lambda = 0$  (no fastidious individuals) this reduces to  $w_L^I > c_H$  which is already part of A4.

We shall not specify the probability of the firm's type since this is irrelevant for the computation of separating equilibria (provided that the types of the individuals are selected independently of the types of the firm). Finally, for simplicity we abstract from discounting between the two periods. This has no effect on our results.

Having outlined the model, we can state four lemmas which play an important role in the analysis. These lemmas state the payoffs of the two types of the firm as functions of  $x$ , for particular first-period prices and consumer beliefs following these prices. These payoffs are used repeatedly in the next section when we outline the requirements for a strategy profile to constitute a separating equilibrium.

**Lemma 1.** *Let  $\pi_{LL}(x)$  be the profits of the low-quality firm, if it spends  $x$  on advertising, sets*

the first-period price at  $w_L^I$ , and every exposed individual in the first-period correctly identifies its type. Then  $\pi_{LL}(x) = 2(1 - \lambda)(w_L^I - c_L)\mu(x) - x$ .

**Proof.** By (1) only indifferent individuals buy in the first period, and the first-period profits are  $(1 - \lambda)(w_L^I - c_L)\mu(x) - x$ . We argue that in the second period the firm sets the same price,  $w_L^I$ . Indeed in the second period the indifferent individuals know the quality of the product. If the second period price is set above  $w_L^I$ , the firm misses out on the purchases of indifferent individuals, and even if by doing so she attracts the fastidious individual, then by A1 and A5 she still obtains less than under the price  $w_L^I$ . Therefore, every individual who observes the second price,  $w_L^I$ , knows that the quality is low. As for prices below  $w_L^I$ , note that the firm can attract the fastidious individuals by setting the price  $w_L^F$  (which by A2 is smaller than  $w_L^I$ ). But this price falls short of its cost (see A4). We conclude that the low-quality firm sets the price  $w_L^I$  in the second period, and the second period profits are  $(1 - \lambda)(w_L^I - c_L)\mu(x)$ . Adding the profits in the two periods, we get  $\pi_{LL}(x) = 2(1 - \lambda)(w_L^I - c_L)\mu(x) - x$  as stated. *Q.E.D.*

The proofs of the next three lemmas use similar arguments and they appear in the appendix.

**Lemma 2.** Let  $\pi_{HH}(x)$  be the profits of the high-quality firm, if it spends  $x$  on advertising, sets the first-period price at  $w_H^I$ , and every exposed individual correctly identifies its type. Then  $\pi_{HH}(x) = 2(w_H^I - c_H)\mu(x) - x$ .

**Lemma 3.** Let  $\pi_{LH}(x)$  be the profits of the low-quality firm if it spends  $x$  on advertising, sets the first-period price at  $w_H^I$ , and succeeds in fooling every exposed individual in the first period into believing that the product is of high quality. Then,  $\pi_{LH}(x) = (w_H^I - c_L + (1 - \lambda)(w_L^I - c_L))\mu(x) - x = (w_H^I - w_L^I + (2 - \lambda)(w_L^I - c_L))\mu(x) - x$ .

**Lemma 4.** Let  $\pi_{HL}(x)$  be the profits of the high-quality firm, if it spends  $x$  on advertising,

sets the first period price at  $w_L^I$ , and every exposed individual incorrectly believes in the first period that the quality is low. Then,  $\pi_{HL}(x) = ((1 - \lambda)(w_L^I - c_H) + w_H^I - c_H)\mu(x) - x = -(1 - \lambda)(w_H^I - w_L^I) + (2 - \lambda)(w_H^I - c_H)\mu(x) - x$ .

Note that  $\pi_{tH}(x) > \pi_{tL}(x)$  for all  $x > 0$  and  $t \in \{H, L\}$ , which means that the firm is better off being perceived as a high-quality firm, irrespective of its true type. By A1 and A3, it is immediate that  $\pi_{t\tau}(x)$  is strictly concave,  $t, \tau \in \{H, L\}$ , and since  $\pi_{t\tau}(0) = 0$  and  $\pi_{t\tau}(x) \rightarrow -\infty$  as  $x \rightarrow \infty$ , we conclude that  $\pi_{t\tau}$  has a single peak.

Next we briefly turn to the “full information” or myopic bench-mark. As mentioned above, the term myopic refers to the case where there is no uncertainty regarding the quality offered for sale. It does *not* imply that all consumers are aware of the existence of the product, but whoever is aware of the product knows its quality whether he makes a purchase or not. First, consider the low-quality firm. Under the hypothetical “full information” case, it clearly makes no sense for the low-quality firm to attempt to sell to fastidious individuals since  $w_L^F < c_L$ . It follows that it is optimal for the firm to set the price in both periods equal to the willingness of indifferent individuals to pay for the low quality. That is,  $p^1 = p^2 = w_L^I \equiv p_L^m$ , where  $m$  stands for “myopic”. Given these prices, only the exposed indifferent individuals buy, and the profits of the low-quality firm as a function of  $x$  are

$$\pi_L^m(x) = 2(1 - \lambda)(w_L^I - c_L)\mu(x) - x \equiv \pi_{LL}(x)$$

$\pi_{LL}(x)$  has a unique interior maximizer,  $x_L^m = \arg \max_x \pi_{LL}(x)$ , which satisfies

$$\mu'(x_L^m) = \frac{1}{2(1 - \lambda)(w_L^I - c_L)}$$

provided that  $\pi_L^m(x_L^m) > 0$ .

Next, consider the high-quality firm. By A5, the proportion of indifferent individuals is significant, and the willingness to pay of the fastidious individuals is not compensatively large. Consequently, the high-quality firm makes sure not to miss out on the purchases of the indifferent individuals. Therefore, it sets the price at  $w_H^I$  and sells to both types of individuals. That is,  $p^1 = p^2 = w_H^I \equiv p_H^m$ . The profits as a function of  $x$  are

$$\pi_H^m(x) = 2(w_H^I - c_H)\mu(x) - x \equiv \pi_{HH}(x)$$

This has a unique interior maximizer,  $x_H^m = \arg \max_x \pi_{HH}(x)$ , which satisfies

$$\mu'(x_H^m) = \frac{1}{2(w_H^I - c_H)}$$

provided that  $\pi_H^m(x_H^m) > 0$ .

To ensure that the full information profits are strictly positive for both types of the firm, that is,  $\pi_{tt}(x_t^m) > 0$ ,  $t \in \{H, L\}$ , we add the following assumption.

$$\mathbf{(A6)} \quad \mu'(0) > \max\left\{\frac{1}{2(1-\lambda)(w_L^I - c_L)}, \frac{1}{2(w_H^I - c_H)}\right\}$$

We note that  $p_H^m = w_H^I > w_L^I = p_L^m$ , and for later reference

$$x_H^m > x_L^m \Leftrightarrow \frac{p_H^m - c_H}{p_L^m - c_L} = \frac{w_H^I - c_H}{w_L^I - c_L} > 1 - \lambda \quad (2)$$

To end this section, we can further explain the assumptions with direct reference to (1) and the “full information” bench-mark. Note first that  $c_H < w_L^I$  (see A4) implies that indifferent individuals are potentially valuable to both types of the firm, since their willingness-to-pay exceeds unit costs. This, together with A5, means that a high-quality firm will never choose to become an “up-market” firm serving only a limited segment of potential customers. Under the alternative assumption  $c_H \geq w_L^I$ , A5 would necessarily be violated for any  $\lambda$ , and the high-quality firm would be more likely to simply ignore the indifferent individuals.

Given the assumptions, a distinguishing feature of the “full information” case is that the low-quality firm only serves the indifferent individuals, whereas the high-quality firm serves both types of consumers. We want to retain this as a feature of the equilibrium, when consumer information is incomplete. Hence, the assumption  $w_L^F < c_L < c_H$  (see A4), that is, the low-quality firm is unable to make money from fastidious individuals once its identity has been revealed.

In terms of descriptive relevance, (1) is intended to capture the following. First of all, consumers are heterogenous. Secondly, there two distinct classes of consumers. This could be thought of as, e.g., *existing* users and *new* users of an established type of product (cars, flat screen TVs, DVD equipment, digital scanners, etc.) or *professional* and *ordinary* users. In the former case, existing users might only be willing to pay a very modest amount,  $w_L^F$ , for a new version that adds modest value (a “low-quality” update),<sup>18</sup> whereas they might be willing to pay a substantial amount,  $w_H^F$ , for a significantly improved new version (a “high-quality” update). Hence,  $w_H^F - w_L^F$  is relatively large. The willingness of new users to pay for the high quality,  $w_H^I$ , and low quality,  $w_L^I$ , may be less sensitive to its exact quality, since they mainly pay for the basic service offered by both versions. So,  $w_H^I - w_L^I$  is relatively small. This rather naturally leads to our assumption A2.

### 3. Separating equilibrium

We study sequential pure strategy separating equilibria (SSE), where the two types of the firm separate themselves in the first period. That is, the signals  $(p_L^1, x_L)$  and  $(p_H^1, x_H)$  are different by at least one component, where  $(p_t^1, x_t)$  is the first-period price and advertising expenditures of the  $t$ -type firm,  $t \in \{H, L\}$ . Furthermore, to sharpen the results we focus on equilibrium

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<sup>18</sup>Note that this could, indeed, be very modest, since existing users of an older version have to subtract the value of the services provided by the old version.

points which are consistent with the Intuitive Criterion of Cho and Kreps (1987). We denote them by SSSE (sensible sequential pure-strategy separating equilibria).

In every separating equilibrium, exposed individuals identify the quality of the product in the first period. They infer that it is of high quality, if they observe the signal  $(p_H^1, x_H)$ , and that it is of low quality, if the signal is  $(p_L^1, x_L)$ . It follows immediately that the low-quality firm chooses the full information price,  $p_L^1 = p_L^m = w_L^I$ , and advertising expenditures,  $x_L = x_L^m$ .

We show below (see Proposition 3) that for a certain region of the parameters, the price of the high-quality firm is  $p_H^1 = p_H^m = w_H^I$  in every SSSE. That is, the price of the high-quality firm is undistorted and coincides with the full information price. But, we first find the advertising expenditures of the high-quality firm associated with an SSSE *under the assumption that  $p_H^1 = w_H^I$* .

Consider the set  $X$ , consisting of all  $x$  such that the low-quality firm finds it unattractive to spend  $x$  on advertising, even if by doing so it succeeds in fooling the exposed individuals about its type. Note that the low-quality firm may fool the consumers only if it sets the first-period price at  $p_H^1 = w_H^I$ . This is the case in  $\pi_{LH}(x)$ . Thus,

$$x \in X \text{ iff } \pi_{LL}(x_L^m) \geq \pi_{LH}(x), \quad (3)$$

(see Lemmas 1 and 3 above).

Suppose now that  $y \notin X$ . Then, in every SSE, the fastidious individuals who are exposed to the product are willing to pay at most  $w_L^F$  and the indifferent individuals are willing to pay at most  $w_L^I$ . Otherwise, if their strategy instructs them to purchase the product at  $w_H^I$ , the low-quality firm is better off switching from  $x_L^m$  to  $y$  (and from  $w_L^I$  to  $w_H^I$ ) since  $\pi_{LH}(y) > \pi_{LL}(x_L^m)$ . Hence, for every advertising expenditure  $y \notin X$ , the high-quality firm sets the price  $w_L^I$  (see (1)

and A5) and obtains the payoff  $\pi_{HL}(y)$ . Therefore,  $x_H$  must satisfy

$$\pi_{HH}(x_H) \geq \max_{y \notin X} \pi_{HL}(y), \quad (4)$$

(see Lemmas 2 and 4 above). That is, the high-quality firm should not be better off at any point where the exposed individuals wrongly identify its type. Finally, at every SSSE, the high-quality firm considers all advertising levels in  $X$  where individuals can “sensibly”<sup>19</sup> infer its identity and selects the one that yields the highest profits. That is,

$$x_H \in \arg \max_{x \in X} \pi_{HH}(x). \quad (5)$$

Conditions (4) and (5) are the incentive-compatibility constraints on the high-quality firm.

Next, we characterize the set  $X$  defined in (3).

**Lemma 5.** *There exist  $\underline{x}$  and  $\bar{x}$  such that:*

- (i)  $0 < \underline{x} < x_L^m < \bar{x}$  and
- (ii)  $X = [0, \underline{x}] \cup [\bar{x}, \infty)$ .

**Proof.** See the Appendix.

Having thus characterized the set,  $X$ , of advertising expenditures that are dominated from the point of view of the low-quality firm, our next goal is to establish conditions to ensure that  $x_H = \underline{x} < x_L^m = x_L$  at every SSSE where  $p_H^1 = w_H^I$ . These are conditions to allow *modest* advertising expenditures to signal high quality. In this section we *first* find conditions which guarantee that  $x_H = \underline{x}$  or  $x_H = x_H^m$  (Lemma 6). *Secondly*, we find conditions for which the incentive-compatibility constraints of the high-quality firm are satisfied for  $x_H = \underline{x}$  (Propositions 1 and 2). *Thirdly*, we show that  $p_H^1 = w_H^I$  will, indeed, be the price chosen by the high-quality

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<sup>19</sup>Since every  $x \in X$  is inferior to  $x_L^m$  for the low-quality firm, it is sensible to infer, when observing  $x \in X$ , that it was selected by the high-quality firm. This inference follows from the Intuitive Criterion of Cho and Kreps (1987) or the forward induction principle of Kohlberg and Mertens (1986).

firm under the conditions stated (Proposition 3). *Finally*, in the next section, where the focus is on signal “reversal”, we strengthen these conditions to guarantee that  $x_L^m < x_H^m$ . Therefore,  $x_H = \underline{x} < x_L = x_L^m < x_H^m$ .

We first find conditions which guarantee that at every SSSE with  $p_H^1 = w_H^I$ , the advertising expenditures of the high-quality firm are smaller than those of the low-quality firm.

**Lemma 6.** *Suppose that  $p_H^1 = w_H^I$ .*

(i) *Then the inequality  $\pi_{HH}(\underline{x}) > \pi_{HH}(\bar{x})$  is equivalent to  $\frac{w_H^I - c_H}{w_L^I - c_L} < 1 - \lambda + \frac{c_H - c_L}{w_L^I - c_L}$ .*

(ii) *If  $\pi_{HH}(\underline{x}) > \pi_{HH}(\bar{x})$ , then  $x_H < x_H^m$ . Furthermore,  $x_H = \underline{x}$  iff  $\underline{x} < x_H^m$ , otherwise  $x_H = x_H^m$ .*

**Proof.** The proof of (i) appears in the Appendix. As for part (ii), the inequality  $\pi_{HH}(\underline{x}) > \pi_{HH}(\bar{x})$  and the concavity of  $\pi_{HH}(x)$  imply that  $x_H^m$  (the maximizer of  $\pi_{HH}(x)$ ) is smaller than  $\bar{x}$ . By (5) and the concavity of  $\pi_{HH}(x)$ ,  $x_H = \underline{x}$  iff  $\underline{x} < x_H^m$ ; otherwise,  $x_H = x_H^m$ . *Q.E.D.*

Lemma 6 implies that whenever the (unit) mark-up of the high-quality firm is not too large compared to the mark-up of the low-quality firm, then the high-quality firm obtains a higher profit under  $\underline{x}$  than under  $\bar{x}$ . Hence, in that case, the high-quality firm prefers to separate at or below  $\underline{x}$  as opposed to separating at or above  $\bar{x}$ .

What remains in this section is to check the incentive-compatibility condition of the high-quality firm given by (4) when  $\frac{w_H^I - c_H}{w_L^I - c_L} < 1 - \lambda + \frac{c_H - c_L}{w_L^I - c_L}$ . The latter can be written as  $2(c_H - c_L) - (w_H^I - w_L^I) - \lambda(w_L^I - c_L) > 0$  which requires  $(w_H^I - w_L^I) < 2(c_H - c_L)$  for any  $\lambda \geq 0$ .

To facilitate the exposition, let us first consider the *special* case without fastidious individuals,  $\lambda = 0$ . In this case the last inequality in (i) of Lemma 6 reduces to  $(w_H^I - w_L^I) < 2(c_H - c_L)$ . Thus, for  $\pi_{HH}(\underline{x}) > \pi_{HH}(\bar{x})$ , the increase in willingness-to-pay for high quality must be less than twice the associated increase in unit costs. We can state the following result.

**Proposition 1.** Suppose that  $p_H^1 = w_H^I$ . If  $\lambda = 0$  and  $w_H^I - w_L^I < 2(c_H - c_L)$ , then the incentive-compatibility condition for the high-quality firm,  $\pi_{HH}(\underline{x}) > \pi_{HL}(x)$ , holds for all  $x \in [\underline{x}, \bar{x}]$ . Further, in every SSSE,  $p_L^1 = w_L^I$ ,  $x_L = x_L^m$ , and  $x_H = \min\{\underline{x}, x_H^m\}$ .

**Proof.** See the Appendix.

Thus, for this case, incentive-compatibility for the high-quality firm is a direct consequence of the assumption  $\frac{w_H^I - c_H}{w_L^I - c_L} < 1 - \lambda + \frac{c_H - c_L}{w_L^I - c_L} = 1 + \frac{c_H - c_L}{w_L^I - c_L}$ , and we need no reference to the properties of  $\mu(x)$  beyond those captured by assumption A6.

For the *general* case with fastidious individuals,  $\lambda \geq 0$ , we need further assumptions about  $\mu(x)$  in the relevant range  $x \in [0, \bar{x}]$ . Heuristically, what is required is that  $\pi_{HL}(x) = ((1 - \lambda)(w_L^I - c_H) + w_H^I - c_H)\mu(x) - x$  should be sufficiently small. That is, the payoffs to a high-quality firm which is mistaken for a low-quality firm in the first period should be small. Otherwise there exist advertising expenditures  $x^* \in [\underline{x}, \bar{x}]$  such that  $\pi_{HL}(x^*) > \pi_{HH}(\underline{x})$ , and incentive-compatibility will be violated. In Section 5 below we provide a fully specified numerical example where  $\mu(x) = 1 - e^{-\alpha x}$ ,  $\alpha > 0$ , that captures this in terms of conditions on  $\alpha$ .

Here, we shall formulate a simple sufficient condition in terms only of  $\mu'(\underline{x})$  and the parameters of the model. We can state the following result.

**Proposition 2.** Suppose that  $p_H^1 = w_H^I$  and  $\lambda \geq 0$ . If

$$(i) \frac{w_H^I - c_H}{w_L^I - c_L} < 1 - \lambda + (1 - \lambda) \frac{c_H - c_L}{w_L^I - c_L} \text{ and}$$

$$(ii) \mu'(\underline{x}) < \frac{1}{(1 - \lambda)(w_L^I - c_H) + (w_H^I - c_H)},$$

then the incentive-compatibility condition for the high-quality firm,  $\pi_{HH}(\underline{x}) > \pi_{HL}(x)$ , holds for all  $x \in [\underline{x}, \bar{x}]$ , and in every SSSE,  $p_L^1 = w_L^I$ ,  $x_L = x_L^m$ , and  $x_H = \min\{\underline{x}, x_H^m\}$ .

**Proof.** For (i) we refer to the text below. For (ii), if  $\mu'(\underline{x}) < \frac{1}{(1 - \lambda)(w_L^I - c_H) + (w_H^I - c_H)}$ , then  $\pi'_{HL}(\underline{x}) < 0$  (by Lemma 4). Thus, by the strict concavity of  $\mu(x)$ , if hypothetically perceived as a

low-quality firm in the first period, the high-quality firm would choose an advertising expenditure  $x^*$  less than  $\underline{x}$ , and it follows that  $\pi_{HL}(x^*) < \pi_{HH}(x^*) \leq \max_{x \in X} \pi_{HH}(x) = \pi_{HH}(x_H)$ . *Q.E.D.*

By assumption A6,  $\mu'(x_L^m) = \frac{1}{2(1-\lambda)(w_L^I - c_L)}$  and  $\underline{x} < x_L^m$ . Thus, for consistency with (ii) of Proposition 2, we require

$$\frac{1}{2(1-\lambda)(w_L^I - c_L)} = \mu'(x_L^m) < \mu'(\underline{x}) < \frac{1}{(1-\lambda)(w_L^I - c_H) + (w_H^I - c_H)}$$

It follows that a necessary condition is  $\frac{w_H^I - c_H}{w_L^I - c_L} < 1 - \lambda + (1 - \lambda)\frac{c_H - c_L}{w_L^I - c_L}$ , which is just condition (ii) of the proposition. This is stronger than  $\frac{w_H^I - c_H}{w_L^I - c_L} < 1 - \lambda + \frac{c_H - c_L}{w_L^I - c_L}$  from Lemma 6 whenever  $\lambda > 0$ , but still perfectly consistent with the general specifications of the model.

Propositions 1 and 2 provide sufficient conditions for the high-quality firm to credibly signal its type by cutting its advertising level below the equilibrium level of the low-quality firm. These results are obtained under the assumption that  $p_H^1 = w_H^I$ . The next result establishes that  $p_H^1 = w_H^I$  will indeed be chosen in every (candidate for) “sensible” separating equilibrium under the stated conditions in Proposition 1 and 2.

**Proposition 3.** *Suppose that  $\frac{w_H^I - c_H}{w_L^I - c_L} < 1 - \lambda + \frac{c_H - c_L}{w_L^I - c_L}$  and  $\pi_{HH}(\underline{x}) > \pi_{HL}(x)$ ,  $\forall x \in [\underline{x}, \bar{x}]$ . Then  $p_H^1 = w_H^I$  in every SSSE.*

**Proof.** *See the Appendix.*

Thus, if the incentive-compatibility condition holds for the relevant advertising expenditures, and if the ratio of the full information mark-ups is as required for modest advertising to be preferred by the high-quality firm, then the high-quality firm will choose the full information price in the first period. The detailed proof appears in the Appendix, but we can briefly sketch the three steps. *First*, we prove that  $p_H^1 \leq w_H^I$ . This follows from  $\lambda(w_H^F - c_H) < (1 - \lambda)(w_L^I - c_H)$  (see A5), which implies that the proportion of indifferent individuals is significant, and the

willingness to pay of the fastidious individuals is not compensatively large. Consequently, by setting  $p_H^1 \leq w_H^I$  the high-quality firm makes sure not to miss out on the purchases of indifferent individuals. *Next*, we use the first condition of the proposition, which asserts that the mark-up of the high-quality firm is not too large compared to that of the low-quality firm. This is used to show that the high-quality firm has no incentive to reduce its price below  $w_H^I$  (to some  $p \in (w_L^F, w_H^I)$ ) and at the same time to increase its advertising expenditures. The associated increase in the number of buyers (present and future) will not offset the first-period decline in mark-up and the additional advertising cost. *Finally*, using the inequality  $w_L^F < c_H$  (see (1)), we show that the high-quality firm has no incentive to reduce its price in the first period to  $w_L^F$  or less, irrespectively of the advertising expenditures. Consequently,  $p_H^1 = w_H^I$  as claimed.

#### 4. The single-crossing condition and signal reversal

The single-crossing condition (SCC), which is standard in the signaling literature, is mainly introduced to guarantee the existence of a separating equilibrium. It may however lead to the exclusion of certain interesting phenomena.<sup>20</sup> Heuristically, the SCC asserts that more aggressive signals are costlier for the weak-type (low-quality) sender than for the strong-type (high-quality) sender, even if the weak-type sender succeeds in fooling the receiver about his type by sending an aggressive signal. The SCC guarantees that in a “sensible” separating equilibrium, the action of the strong-type sender is more aggressive than that of the weak-type sender. In our context, for the case where  $x_H^m > x_L^m$ , this would mean more generous advertising in the signaling equilibrium by the high-quality firm (contrary to Propositions 1 and 2 above), that is,  $x_H > x_H^m > x_L^m = x_L$ . Thus, the advertising expenditures of the high-quality firm would be distorted *above* the full information level, in the case where this is higher than the full information level of the low-quality

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<sup>20</sup> Bagwell and Bernheim (1996) observed that while “...Veblen effects cannot ordinarily arise when preferences satisfy a single-crossing property, they may emerge when this property fails.”

firm.

In our model, the SCC is equivalent to the requirement that  $\pi_{HH}(x) - \pi_{LH}(x)$  is a monotone increasing function of  $x$  when the parameters are such that  $x_H^m > x_L^m$ . We claim that this is not always the case, especially if  $c_H$  is sufficiently large relative to  $c_L$ . The reason is as follows: When the low-quality firm mimics the high-quality firm, it sets the high-quality price. Therefore, the low-quality firm has the higher mark-up ( $w_H^I - c_L$  compared to  $w_H^I - c_H$ ). Thus, the low-quality firm has the advantage that it profits from every unit sold more than the high-quality firm. On the other hand, for every additional unit sold in the first period, the high-quality firm sells more than the low-quality firm in the second period, due to repeat-purchases. This advantage of the high-quality firm from large advertising expenditures vanishes as  $(w_H^I - c_H)/(w_L^I - c_L)$  becomes sufficiently small.

First, we strengthen the conditions of Proposition 1 and 2 to allow for the signal reversal phenomenon.<sup>21</sup> We start with a lemma which follows immediately from (2).

**Lemma 7.** *Suppose that  $\frac{w_H^I - c_H}{w_L^I - c_L} > 1 - \lambda$ . Then  $x_L^m < x_H^m$ , and  $x_H \geq \underline{x}$ .*

Since the profits of the high-quality firm,  $\pi_{HH}(x)$ , are increasing in  $x$  for  $x \leq x_H^m$ , it follows that  $x_H \geq \underline{x}$  for  $x_L^m < x_H^m$ .

For the *special* case without fastidious individuals,  $\lambda = 0$ , the condition in Lemma 7 reduces to  $w_H^I - w_L^I > c_H - c_L$ . Hence, we can combine Lemma 7 and Propositions 1 and 3 to state the following result.

**Theorem 1 (Signal Reversal,  $\lambda = 0$ ).** *Suppose that  $\lambda = 0$  and  $w_H^I - w_L^I \in (c_H - c_L, 2(c_H - c_L))$ . Then there exists a unique SSSE outcome with the following properties:*

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<sup>21</sup>Signal reversal in the context of two senders was introduced in Orzach and Tauman (1996)

$$p_L^1 = p_L^2 = w_L^I, p_H^1 = p_H^2 = w_H^I, x_L = x_L^m, x_H = \underline{x} \text{ and } \underline{x} < x_L^m < x_H^m.$$

**Proof.** The proof follows immediately from Propositions 1 and 3 and Lemma 7. *Q.E.D.*

For the general case with fastidious individuals, we can combine Lemma 7 and Propositions 2 and 3 into the following result.

**Theorem 2 (Signal Reversal,  $\lambda > 0$ ).** Suppose that

- (i)  $1 - \lambda < \frac{w_H^I - c_H}{w_L^I - c_L} < 1 - \lambda + (1 - \lambda) \frac{c_H - c_L}{w_L^I - c_L}$  and
- (ii)  $\mu'(\underline{x}) < \frac{1}{(1-\lambda)(w_L^I - c_H) + (w_H^I - c_H)}$ .

Then there exists a unique SSSE outcome with the following properties:

$$p_L^1 = p_L^2 = w_L^I, p_H^1 = p_H^2 = w_H^I, x_L = x_L^m, x_H = \underline{x} \text{ and } \underline{x} < x_L^m < x_H^m.$$

**Proof.** The proof follows immediately from Propositions 2, 3 and Lemma 7. *Q.E.D.*

Note that by the concavity of  $\mu(x)$ , we could replace  $\mu'(\underline{x})$  in (ii) by  $\mu'(0)$  to obtain a global but more restrictive condition.

Also, at this point the reader should recall that condition (ii) of Theorem 2 is a sufficient condition. In the example below we present a case where this condition is not satisfied, yet high quality is still signalled by a modest advertising level.

The upshot of these two results is that the conditions for signal reversal can be satisfied in a proper region of the parameters.<sup>22</sup> The ratio of the full information mark-ups,  $\frac{w_H^I - c_H}{w_L^I - c_L}$ , should neither be too small nor too large. If the ratio is too small, then  $x_H^m < x_L^m$ , and we do not have signal reversal in the sense of this paper. If the ratio is too large, we have  $x_H^m > x_L^m$  and an upward distortion of advertising expenditures by the high-quality firm (if anything). Clearly, a necessary condition for signal reversal in the sense of this paper is that the ratio of the mark-ups

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<sup>22</sup>We provide an explicit example in the next section.

be in an *intermediate* range  $\frac{w_H^I - c_H}{w_L^I - c_L} \in (1 - \lambda, 1 - \lambda + \frac{c_H - c_L}{w_L^I - c_L})$ . An immediate corollary is the following.

**Corollary 1.** *If  $\lambda$  is sufficiently small, then signal reversal is consistent with mark-ups which are positively related to quality in the full information case, that is,  $\frac{w_H^I - c_H}{w_L^I - c_L} > 1$ .*

Thus, if the fraction of “fastidious” individuals is not too large, then in a proper region of the parameters the model is consistent with the phenomenon of signal reversal *and* equilibrium mark-ups which are increasing in quality:

Indeed, in the *special* case without fastidious individuals,  $x_L^m < x_H^m$  is equivalent to mark-ups which are increasing in quality, and in this case increasing mark-ups are associated with modest advertising in equilibrium provided that  $w_H^I - w_L^I \in (c_H - c_L, 2(c_H - c_L))$ .

Finally, we show that signal reversal is incompatible with the SCC. Recall that signal “reversal” in an SSSE requires  $\pi_{HH}(\underline{x}) > \pi_{HH}(\bar{x})$  and  $x_H^m > x_L^m$ .

**Proposition 4.** *The SCC does not hold in any SSSE where  $\pi_{HH}(\underline{x}) > \pi_{HH}(\bar{x})$  and  $x_H^m > x_L^m$ .*

**Proof.** See the Appendix.

The proof shows that if  $\pi_{HH}(x) - \pi_{LH}(x)$  is a monotone increasing function of  $x$  and the parameters are such that  $x_H^m > x_L^m$ , then  $\pi_{HH}(\underline{x}) < \pi_{HH}(\bar{x})$ . Hence, SCC together with  $x_H^m > x_L^m$  would lead to the standard result: the high-quality firm signals by increasing introductory advertising expenditures above the full information level (if anything).

## 5. Example

To highlight the scope for signaling with a *modest* amount of advertising in the *general* case with fastidious individuals,  $\lambda > 0$ , we shall give a fully specified and robust example which gives

rise to signal reversal. The example considers two cases. In the first case, both conditions in Theorem 2 are satisfied. In the other case the second condition of Theorem 2 is violated. This is meant to illustrate that this condition is sufficient but not necessary (as we remarked in the discussion above).

Assume that  $\mu(x) = 1 - e^{-\alpha x}$ ,  $\alpha > 0$ , which is increasing, strictly concave with  $\mu(0) = 0$  and  $\mu(x) \rightarrow 1$  as  $x \rightarrow \infty$ . Also,  $\mu'(x) = \alpha e^{-\alpha x}$  and  $\mu'(0) = \alpha$ . Further, fix the valuations and costs as follows:  $w_L^I = 4.5$ ,  $w_H^I = 6$ ,  $w_L^F < 1.5$ ,  $w_H^F \in (6, 18.5)$ ,  $c_L = 1.5$ ,  $c_H = 2.75$ . This immediately implies that  $\frac{w_H^I - c_H}{w_L^I - c_L} = \frac{13}{12} > 1 > 1 - \lambda$ . Hence, in the example we assume at the outset that the full information mark-ups are increasing in quality. Finally, assume that  $\lambda = \frac{1}{10}$ . Given this specification, it is easy to demonstrate that assumptions A1 through A5 are satisfied. As for assumption A6, we require  $\alpha > \frac{5}{27}$ , and we shall assume this from now on. To complete the description, we note that the payoffs are as follows;  $\pi_{LL}(x) = \frac{27}{5} - \frac{27}{5}e^{-\alpha x} - x$ ,  $\pi_{HH}(x) = \frac{13}{2} - \frac{13}{2}e^{-\alpha x} - x$ ,  $\pi_{LH}(x) = \frac{72}{10} - \frac{72}{10}e^{-\alpha x} - x$  and  $\pi_{HL}(x) = \frac{193}{40} - \frac{193}{40}e^{-\alpha x} - x$ .

*Case 1:  $\alpha = \frac{1}{5}$*

Rather than working out all the details of the example (for that, see Case 2 below), we simply note that if  $\alpha = \frac{1}{5}$ , then

$$\mu'(\underline{x}) < \mu'(0) = \alpha = \frac{1}{5} < \frac{40}{193} = \frac{1}{(1 - \lambda)(w_L^I - c_H) + (w_H^I - c_H)}$$

That is, condition (ii) of Theorem 2 is satisfied. Hence,  $\pi'_{HL}(x) = \frac{193}{40}\alpha e^{-\alpha x} - 1 = \frac{193}{200}e^{-x/5} - 1 < 0$ , and the maximizer of  $\pi_{HL}(x)$  is  $x^* = 0 < \underline{x}$ . It follows immediately that we have signal reversal in this case:  $x_H = \underline{x} < x_L = x_L^m < x_H^m$ .

*Case 2:  $\alpha = \frac{1}{4}$*

In this case the full information payoffs of the low-quality firm are  $\pi_{LL}(x) = \frac{27}{5} - \frac{27}{5}e^{-x/4} - x$ , and the maximizer is  $x_L^m = 1.2004$ , which in turn implies that  $\pi_{LL}(x_L^m) = 0.19958$ . Similarly,

the full information payoffs of the high-quality firm are  $\pi_{HH}(x) = \frac{13}{2} - \frac{13}{2}e^{-x/4} - x$ , and the maximizer is  $x_H^m = 1.942 > 1.2004 = x_L^m$ , while the profits are  $\pi_{HH}(x_H^m) = 0.55797$ .

To determine  $\underline{x}$  and  $\bar{x}$ , we note that  $\pi_{LH}(x) = \frac{72}{10} - \frac{72}{10}e^{-x/4} - x$ . We solve  $\pi_{LH}(x) = \pi_{LL}(x_L^m) = 0.19958$  to obtain  $\underline{x} = 0.26944$  and  $\bar{x} = 4.8688$ . A high-quality firm which is mistaken for a low-quality firm in the first period obtains  $\pi_{HL}(x) = \frac{193}{40} - \frac{193}{40}e^{-x/4} - x$ , and the maximizer is  $x^* = 0.75007 > \underline{x}$  with associated profits  $\pi_{HL}(x^*) = 0.074935$ . Thus, in this case the high-quality firm would choose a level of advertising in excess of  $\underline{x}$ , if he were mistakenly thought to have a low quality in the first period. Hence, the example violates the sufficient condition in Theorem 2. To see this directly, we note that  $\mu'(\underline{x}) = \frac{1}{4}e^{-\underline{x}/4} = 0.23371 > 0.20725 = \frac{40}{193} = \frac{1}{(1-\lambda)(w_L^I - c_H) + (w_H^I - c_H)}$ .

Now,  $\pi_{HH}(\underline{x}) = 0.15398 > \pi_{HL}(x^*) = 0.074935 > \pi_{HH}(\bar{x}) = -0.82359$ . Hence, the incentive-compatibility condition for the high-quality firm is satisfied and all the conditions for signal reversal are satisfied despite the violation of condition (ii) of Theorem 2. Thus, we have signal reversal:  $x_H = \underline{x} < x_L = x_L^m < x_H^m$ .<sup>23</sup>

So, with up to at least 10% fastidious individuals (including the special case with no fastidious individuals at all), the example is certainly consistent with mark-ups which are *positively* related to quality and *modest* advertising as a signal of high quality.

*Remarks.* For the *special* case with homogenous consumers, an empirical test “merely” requires *ex post* information on advertising expenditures, prices, and measures of unit cost across a sample of products and qualities. If mark-ups increase rapidly with quality, we expect high qualities to be associated with large advertising expenditures, while if mark-ups increase moderately (or decrease) with quality, we expect high qualities to be associated with modest

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<sup>23</sup>Increasing  $\alpha$  above  $\frac{1}{4}$  will increase  $x^*$  compared to  $x_L^m$ , and for  $\alpha$  sufficiently large the incentive-compatibility condition of the high-quality firm will be violated.

advertising expenditures. In addition to the information needed for the special case, empirical testing of the *general* case with heterogenous consumers requires information on consumers allowing a separation of two types, such as *experienced* and *first-time* buyers or *professional* and *private* users.

A couple of more general comments might be in order at this point. First, it would be valuable in terms of descriptive relevance to extend the modelling of consumer heterogeneity to the case with multiple classes of consumers. Secondly, the model of this paper shares with the existing literature on quality signaling the feature that the firm has a monopoly. Very little theoretical work has been done on quality signaling by competing oligopolistic firms. This poses a severe problem for empirical testing based on data collected from oligopolistic industries, since it is unclear whether the main qualitative conclusions for the monopoly case survive the introduction of competition between firms.<sup>24</sup>

## 6. Concluding remarks

To summarize, we have shown that a high-quality firm may, indeed, choose to signal its identity by lowering introductory advertising expenditures to a level below that of the low-quality firm. We refer to this as signal reversal, if under full information the high-quality firm would have chosen a higher level of advertising than the low-quality firm. Also, this phenomenon is consistent with a positive relationship between the equilibrium mark-ups and quality. This requires that the ratio between these mark-ups be neither too small nor too large, and that the fraction of potential customers who are highly sensitive to quality is not too large.

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<sup>24</sup>See Hertzendorf and Overgaard (2001) for an analysis of quality signaling by competing firms.

## Appendix

Proofs of Lemma 2 - 6 and Proposition 1, 3 and 4 follow.

*Proof of Lemma 2.* By (1) in the text, all those exposed buy in the first period, and first-period profits are  $(w_H^I - c_H)\mu(x) - x$ . In the second period, first-period buyers (all those exposed) know the quality. Therefore, the firm sets the same price,  $w_H^I$ , every first-period buyer makes a repeat purchase, and the second-period profits are  $(w_H^I - c_H)\mu(x)$ . Adding, we get  $\pi_{HH}(x) = 2(w_H^I - c_H)\mu(x) - x$ . *Q.E.D.*

*Proof of Lemma 3.* By (1), all those exposed buy in the first period, and first-period profits are  $(w_H^I - c_L)\mu(x) - x$ . In the second period all those exposed know the quality, by (1) the optimal second-period price is  $w_L^I < w_H^I$ , and only the indifferent individuals make a repeat purchase. Second-period profits are  $(1 - \lambda)(w_L^I - c_L)\mu(x)$ . Adding, we get  $\pi_{LH}(x) = (w_H^I - w_L^I + (2 - \lambda)(w_L^I - c_L))\mu(x) - x$ . *Q.E.D.*

*Proof of Lemma 4.* By (1) only the indifferent individuals among those exposed buy in the first period, and first-period profits are  $(1 - \lambda)(w_L^I - c_H)\mu(x) - x$ . Next, we argue that the optimal second-period price is  $w_H^I > w_L^I$ , and all those initially exposed buy in the second period. First observe that every exposed indifferent individual knows (by experience) that quality is high and is therefore willing to pay  $w_H^I$ . The exposed fastidious individuals (who have observed the first-period price  $w_L^I$ ) *must* infer that quality is high, if the second-period price is higher than the first-period price. To see this, recall that they know that the indifferent individuals know the quality by experience and will be willing to pay a price higher than  $w_L^I$  only for a high quality product. By assumption A5 in the text, the firm can not afford to miss out on the purchases of the indifferent individuals. Therefore, it will raise the price above  $w_L^I$  *only if* it produces high quality.

Thus, the fastidious individuals who were initially exposed identify the type of the firm and, consequently, will purchase the product, if the second-period price is  $w_H^I$ . Clearly, by assumption A5, the firm has no incentive to raise the price above  $w_H^I$  to  $w_H^F$ . Consequently, the firm sets the price  $w_H^I$ , and the second-period profits are  $(w_H^I - c_H)\mu(x)$ . Adding, we get  $\pi_{HL}(x) = -(1 - \lambda)(w_H^I - w_L^I) + (2 - \lambda)(w_H^I - c_H)\mu(x) - x$ . *Q.E.D.*

*Proof of Lemma 5.* By (1) and assumption A3 in the text,  $\pi_{LH}(x)$  is a strictly concave function with  $\pi_{LH}(0) = 0$  and  $\pi_{LH}(x) \rightarrow -\infty$  for  $x \rightarrow \infty$ . In addition,  $\pi_{LL}(x) < \pi_{LH}(x)$ ,  $\forall x > 0$ . Together with assumption A6, this implies  $\pi_{LH}(0) = 0 < \pi_{LL}(x_L^m) < \pi_{LH}(x_L^m)$ . Thus, by the continuity of  $\pi_{LH}(x)$ , there exists  $\underline{x}$ ,  $0 < \underline{x} < x_L^m$  such that  $\pi_{LH}(\underline{x}) = \pi_{LL}(x_L^m)$ . Similarly,  $\pi_{LH}(x_L^m) > \pi_{LL}(x_L^m) > \lim_{x \rightarrow \infty} \pi_{LH}(x) = -\infty$ . Therefore, there exists  $\bar{x}$ ,  $\bar{x} > x_L^m$  such that  $\pi_{LH}(\bar{x}) = \pi_{LL}(x_L^m) = \pi_{LH}(\underline{x})$ . By the concavity of  $\pi_{LH}(x)$ , the set  $X$  defined in (3) is of the form  $X = [0, \underline{x}] \cup [\bar{x}, \infty)$ . *Q.E.D.*

*Proof of Lemma 6.* We prove part (i). By Lemma 3,  $\pi_{LH}(\underline{x}) = \pi_{LH}(\bar{x})$  is equivalent to

$$\bar{x} - \underline{x} = (2 - \lambda)(w_L^I - c_L)(\mu(\bar{x}) - \mu(\underline{x})) + (w_H^I - w_L^I)(\mu(\bar{x}) - \mu(\underline{x}))$$

which we can write as

$$\bar{x} - \underline{x} = [(1 - \lambda)(w_L^I - c_L) + (w_H^I - c_H) + (c_H - c_L)](\mu(\bar{x}) - \mu(\underline{x})) \quad (\text{A1})$$

Also, note that by Lemma 2

$$\pi_{HH}(\underline{x}) \geq \pi_{HH}(\bar{x}) \text{ iff } \bar{x} - \underline{x} \geq 2(w_H^I - c_H)(\mu(\bar{x}) - \mu(\underline{x})) \quad (\text{A2})$$

Hence, by (A1) and (A2),  $\pi_{HH}(\underline{x}) \geq \pi_{HH}(\bar{x})$  iff

$$(1 - \lambda)(w_L^I - c_L) + (w_H^I - c_H) + (c_H - c_L) \geq 2(w_H^I - c_H)$$

which can be written as  $\frac{w_H^I - c_H}{w_L^I - c_L} \leq 1 - \lambda + \frac{c_H - c_L}{w_L^I - c_L}$ . *Q.E.D.*

*Proof of Proposition 1.* We start by considering the general case,  $\lambda \geq 0$ , and then specialize to  $\lambda = 0$  when appropriate. So, suppose that  $\frac{w_H^I - c_H}{w_L^I - c_L} < 1 - \lambda + \frac{c_H - c_L}{w_L^I - c_L}$ , which we can write as  $2(c_H - c_L) - (w_H^I - w_L^I) - \lambda(w_L^I - c_L) > 0$ . This assumption implies  $x_H^m < \bar{x}$ . Further, we define  $x^* = \arg \max_{x \in \mathfrak{R}_+} \pi_{HL}(x)$ . Now, since  $\pi_{HL}(x) < \pi_{HH}(x)$  for all  $x > 0$ , it is easy to show that  $x^* < x_H^m < \bar{x}$ .

First, if  $x^* \leq \underline{x}$ , then  $\pi_{HH}(\underline{x}) > \pi_{HL}(\underline{x}) > \pi_{HL}(x^*)$ ,  $\forall x \in (\underline{x}, \bar{x}]$ . Hence, the incentive-compatibility condition for the high-quality firm holds in this case.

Next, suppose that  $x^* \in (\underline{x}, x_H^m)$ . Now, incentive-compatibility for the high-quality firm requires

$$\pi_{HH}(\underline{x}) > \pi_{HL}(x^*) \quad (\text{A3})$$

By construction, we have  $\pi_{LH}(\underline{x}) = \pi_{LL}(x_L^m) \equiv \pi_{LL}(x^*) + (\pi_{LL}(x_L^m) - \pi_{LL}(x^*))$ . Using this in (A3), we can rewrite the incentive condition as

$$\pi_{HH}(\underline{x}) - \pi_{LH}(\underline{x}) > \pi_{HL}(x^*) - \pi_{LL}(x^*) - (\pi_{LL}(x_L^m) - \pi_{LL}(x^*))$$

or

$$\pi_{LL}(x^*) - \pi_{HL}(x^*) + \pi_{LL}(x_L^m) - \pi_{LL}(x^*) > \pi_{LH}(\underline{x}) - \pi_{HH}(\underline{x}) \quad (\text{A4})$$

Using Lemmas 1 through Lemma 4, we can generally write  $\pi_{LL}(x) - \pi_{HL}(x) = \pi_{LH}(x) - \pi_{HH}(x) - \lambda(c_H - c_L)\mu(x)$ , which we use in (A4) to obtain

$$\pi_{LH}(x^*) - \pi_{HH}(x^*) - (\pi_{LH}(\underline{x}) - \pi_{HH}(\underline{x})) + \pi_{LL}(x_L^m) - \pi_{LL}(x^*) > \lambda(c_H - c_L)\mu(x^*) \quad (\text{A5})$$

Also, we can write  $\pi_{LH}(x) - \pi_{HH}(x) = (2(c_H - c_L) - (w_H^I - w_L^I) - \lambda(w_L^I - c_L))\mu(x)$ . Applying this to (A5), we can write the incentive-compatibility constraint of the high-quality firm as

$$(2(c_H - c_L) - (w_H^I - w_L^I) - \lambda(w_L^I - c_L))(\mu(x^*) - \mu(\underline{x})) + \pi_{LL}(x_L^m) - \pi_{LL}(x^*) > \lambda(c_H - c_L)\mu(x^*) \quad (\text{A6})$$

By the first condition of Lemma 6,  $2(c_H - c_L) - (w_H^I - w_L^I) - \lambda(w_L^I - c_L) > 0$ , and by assumption,  $\mu(x^*) - \mu(\underline{x}) > 0$ , while by construction,  $\pi_{LL}(x_L^m) - \pi_{LL}(x^*) \geq 0$ . Hence, the left-hand-side of (A6) is strictly positive, and we conclude quite generally that the inequality is satisfied provided that  $\lambda(c_H - c_L)\mu(x^*)$  is sufficiently small.

Turning to the special case with  $\lambda = 0$ , (A6) reduces to

$$(2(c_H - c_L) - (w_H^I - w_L^I))(\mu(x^*) - \mu(\underline{x})) + (\pi_{LL}(x_L^m) - \pi_{LL}(x^*)) > 0$$

But, by the assumption that  $2(c_H - c_L) - (w_H^I - w_L^I) > 0$ , the first term on the left-hand-side is strictly positive. The second term is non-negative, and it follows that incentive-compatibility is satisfied. This completes the proof of Proposition 1. *Q.E.D.*

*Proof of Proposition 3.* The proof is based on three claims.

*Claim 1.*  $p_H^1 \leq w_H^I$

Proof. Suppose to the contrary that  $p_H^1 > w_H^I$  (w.l.o.g. we assume that  $p_H^1 \leq w_H^F$ ). By (1) in the text and Lemma 5, all individuals believe that the firm is of type H whenever they observe  $(p^1, x)$  in the first period such that  $x \in X$  and  $p^1 \geq w_H^I$ . Therefore,  $(p_H^1, x)$  can not be an SSSE signal if  $x \in X$  since the high-quality firm is better off deviating to  $w_H^I$  to attract the indifferent consumers. Suppose, therefore, that  $x \notin X$ . Define for  $p \in (w_H^I, w_H^F]$

$$\Pi_{HH}(p, x) \equiv \lambda(p - c_H)\mu(x) + (w_H^I - c_H)\mu(x) - x$$

The function  $\Pi_{HH}(p, x)$  is the “optimistic” profit of the high-quality firm, if it spends  $x$  on advertising and sets the first-period price at  $p > w_H^I$ . Only the fastidious consumers purchase the product. In the second period, the price is  $w_H^I$ , and every exposed individual correctly identifies the firm’s type and purchases the product. To derive a contradiction we will show that  $\Pi_{HH}(p, x) < \pi_{HH}(\underline{x})$ , which means that the high-quality firm is better off deviating to  $(w_H^I, \underline{x})$ .

By assumption A5 in the text and  $p \in (w_H^I, w_H^F]$ ,

$$\lambda(p - c_H)\mu(x) \leq \lambda(w_H^F - c_H)\mu(x) < (1 - \lambda)(w_L^I - c_H)\mu(x)$$

This implies that

$$\Pi_{HH}(p, x) < ((1 - \lambda)(w_L^I - c_H) + (w_H^I - c_H))\mu(x) - x = \pi_{HL}(x)$$

using the definition of  $\pi_{HL}(x)$  from Lemma 4. Thus, by the second supposition of the proposition,  $\Pi_{HH}(p, x) < \pi_{HL}(x) < \pi_{HH}(\underline{x})$  for all  $x \in [\underline{x}, \bar{x}]$ . This completes the proof of the first claim.

*Claim 2.* In every SSSE,  $p_H^1 \notin (w_L^F, w_H^I)$

*Proof.* For all  $p \in (w_L^F, w_H^I]$  let

$$\Pi_{LH}(p, x) = ((p - c_L) + (1 - \lambda)(w_L^I - c_L))\mu(x) - x \quad (\text{A7})$$

$\Pi_{LH}(p, x)$  is the profit of the low-quality firm, if it spends  $x$  on advertising, sets the first-period price  $p$ , and succeeds in fooling all exposed individuals in the first period when they observe  $(p, x)$ . Note that  $\Pi_{LH}(w_H^I, x) = \pi_{LH}(x)$ . Similarly, for all  $p \in (w_L^F, w_H^I]$  let

$$\Pi_{HH}(p, x) = ((p - c_H) + (w_H^I - c_H))\mu(x) - x \quad (\text{A8})$$

$\Pi_{HH}(p, x)$  is the profit of the high-quality firm, if it chooses the signal  $(p, x)$ , and all exposed individuals correctly identify its type. Note that  $\Pi_{HH}(w_H^I, x) = \pi_{HH}(x)$ .

Now, take any candidate for equilibrium play by the high-quality firm,  $(p_H^1, x_H)$ , with  $w_L^F < p_H^1 < w_H^I$ . First, we prove that in every SSSE,  $p_H^1$  and  $x_H$  must be such that

$$\Pi_{LH}(p_H^1, x_H) = \pi_{LL}(x_L^m) \quad (\text{A9})$$

Indeed, if  $\Pi_{LH}(p_H^1, x_H) < \pi_{LL}(x_L^m)$ , then there exists  $p \in (p_H^1, w_H^I)$  such that

$$\Pi_{LH}(p, x_H) < \pi_{LL}(x_L^m) \quad (\text{A10})$$

By (A10), the action  $(p, x_H)$  is inferior to  $(w_L^I, x_L^m)$  from the point of view of the low-quality firm, and it follows that every exposed individual believes that quality is high when they observe  $(p, x_H)$ . But then the high-quality firm obtains a higher profit from  $(p, x_H)$  than from  $(p_H^1, x_H)$ . This contradicts that  $(p_H^1, x_H)$  is an equilibrium action of the high-quality firm. Thus,  $\Pi_{LH}(p_H^1, x_H) \geq \pi_{LL}(x_L^m)$ . But if  $\Pi_{LH}(p_H^1, x_H) > \pi_{LL}(x_L^m)$ , then the low-quality firm is better off mimicking the high-quality firm, a contradiction. This establishes (A9).

Next, let  $p_H^1 = w_H^I - \epsilon_1$ , with  $\epsilon_1 > 0$  and let  $x_H = \underline{x} + \epsilon_2$ . Suppose, first, that  $\epsilon_2 < 0$ . Then  $x_H < \underline{x}$  and, hence,  $x_H \in X$  (see Lemma 5). All exposed individuals believe that the product is of high quality when they observe  $(w_H^I, x_H)$ . Thus, the profits of the high-quality firm would be higher under  $(w_H^I, x_H)$  than under  $(p_H^1, x_H)$  (since  $w_H^I > p_H^1$ ) which upsets the equilibrium. Suppose, next, that  $\epsilon_2 > \bar{x} - \underline{x}$ . Then,  $x_H > \bar{x}$ , and  $x_H \in X$  (by Lemma 5). Again, this destabilizes the equilibrium. What remains is the case where  $0 \leq \epsilon_2 \leq \bar{x} - \underline{x}$ . By (A9),

$$\Pi_{LH}(w_H^I - \epsilon_1, \underline{x} + \epsilon_2) = \pi_{LL}(x_L^m) = \pi_{LH}(\underline{x}) = \Pi_{LH}(w_H^I, \underline{x}) \quad (\text{A11})$$

By (A7) above,

$$\Pi_{LH}(w_H^I - \epsilon_1, \underline{x} + \epsilon_2) = ((w_H^I - \epsilon_1 - c_L) + (1 - \lambda)(w_L^I - c_L))\mu(\underline{x} + \epsilon_2) - (\underline{x} + \epsilon_2)$$

which we can rewrite as

$$\Pi_{LH}(w_H^I - \epsilon_1, \underline{x} + \epsilon_2) = ((w_H^I - \epsilon_1 - c_H) + (w_H^I - c_H))\mu(\underline{x} + \epsilon_2) - (\underline{x} + \epsilon_2) + R\mu(\underline{x} + \epsilon_2)$$

where

$$R = 2(c_H - c_L) - (w_H^I - w_L^I) - \lambda(w_L^I - c_L)$$

Using (A8), we have

$$\Pi_{LH}(w_H^I - \epsilon_1, \underline{x} + \epsilon_2) = \Pi_{HH}(w_H^I - \epsilon_1, \underline{x} + \epsilon_2) + R\mu(\underline{x} + \epsilon_2) \quad (\text{A12})$$

Now observe that

$$\Pi_{LH}(w_H^I, \underline{x}) = ((w_H^I - c_L) + (1 - \lambda)(w_L^I - c_L))\mu(\underline{x}) - \underline{x}$$

which we can write as

$$\Pi_{LH}(w_H^I, \underline{x}) = ((w_H^I - c_H) + (w_H^I - c_H))\mu(\underline{x}) - \underline{x} + R\mu(\underline{x})$$

Hence, using (A8) again

$$\Pi_{LH}(w_H^I, \underline{x}) = \Pi_{HH}(w_H^I, \underline{x}) + R\mu(\underline{x}) \quad (\text{A13})$$

By (A11),  $\Pi_{LH}(w_H^I - \epsilon_1, \underline{x} + \epsilon_2) = \Pi_{LH}(w_H^I, \underline{x})$ . Thus, by (A12) and (A13),

$$\Pi_{HH}(w_H^I - \epsilon_1, \underline{x} + \epsilon_2) - \Pi_{HH}(w_H^I, \underline{x}) = R(\mu(\underline{x}) - \mu(\underline{x} + \epsilon_2))$$

What remains is to show that  $R(\mu(\underline{x}) - \mu(\underline{x} + \epsilon_2)) < 0$ . Since  $\mu(\underline{x}) - \mu(\underline{x} + \epsilon_2) < 0$ , it is sufficient that  $R = 2(c_H - c_L) - (w_H^I - w_L^I) - \lambda(w_L^I - c_L) > 0$ . But this is just  $\frac{w_H^I - c_H}{w_L^I - c_L} < 1 - \lambda + \frac{c_H - c_L}{w_L^I - c_L}$ , which is the first supposition in the proposition. This completes the proof of the second claim.

*Claim 3.*  $p_H = w_H^I$

Proof. By Claims 1 and 2, it is sufficient to rule out the possibility that  $p_H \leq w_L^F$ . Clearly,  $p_H = w_L^F$  dominates any  $p_H < w_L^F$ , since at  $p_H = w_L^F$  all exposed individuals buy whatever their beliefs are. It remains to rule out  $p_H = w_L^F$ . By assumption A5 in the text and Lemma 5, every exposed individual believes that the firm is of type H whenever they observe  $(w_H^I, x)$  in the first period such that  $x \in X$ . Therefore  $(p_H, x)$  can not be an SSSE signal if  $x \in X$ , since the high-quality firm is better off deviating to  $w_H^I$  where it can still attract all exposed consumers. Suppose therefore that  $x \in (\underline{x}, \bar{x})$  and  $p_H = w_L^F$ .

$\Pi_{HH}(w_L^F, x)$  is the profit of the high-quality firm, if it spends  $x$  on advertising, sets the first-period price at  $w_L^F$  such that all exposed individuals buy in the first period, and optimally

sets the second-period price at  $w_H^I$  such that all exposed individuals buy in the second period.

Then

$$\Pi_{HH}(w_L^F, x) = ((w_L^F - c_H) + (w_H^I - c_H))\mu(x) - x$$

Let us show that  $\Pi_{HH}(w_L^F, x) < \pi_{HL}(x)$ , i.e.,

$$((w_L^F - c_H) + (w_H^I - c_H))\mu(x) - x < ((1 - \lambda)(w_L^I - c_H) + (w_H^I - c_H))\mu(x) - x$$

But this follows immediately from  $w_L^F < c_H < w_L^I$  (see (1)). Hence, by the second supposition of the proposition,  $\Pi_{HH}(w_L^F, x) < \pi_{HL}(x) < \pi_{HH}(\underline{x})$ , and the third claim is established. This completes the proof of Proposition 3. *Q.E.D.*

*Proof of Proposition 4.* Whenever  $x_H^m > x_L^m$ , the SCC condition is equivalent to the requirement that  $\pi_{HH}(x) - \pi_{LH}(x)$  is increasing in  $x$ . By Lemmas 2 and 3,

$$\begin{aligned} \pi_{HH}(x) - \pi_{LH}(x) &= 2(w_H^I - c_H)\mu(x) - x - (w_H^I - c_L + (1 - \lambda)(w_L^I - c_L))\mu(x) + x \\ &= -[2(c_H - c_L) - (w_H^I - w_L^I) - \lambda(w_L^I - c_L)]\mu(x) \end{aligned}$$

But since  $\mu'(x) > 0$ , the monotonicity requirement reduces to  $2(c_H - c_L) - (w_H^I - w_L^I) - \lambda(w_L^I - c_L) < 0$  or  $\frac{w_H^I - c_H}{w_L^I - c_L} > 1 - \lambda + \frac{c_H - c_L}{w_L^I - c_L}$ . However, this reverses the condition needed to ensure  $\pi_{HH}(\underline{x}) > \pi_{HH}(\bar{x})$  (see Lemma 6), and we conclude that the SCC is incompatible with signal reversal. *Q.E.D.*

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