Capital adjustment costs that mimic liquidity constraints in a two period model

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Abstract.- In a two-period deterministic model with capital as the only factor, adjustment cost functions induce liquidity constrained investment when they reproduce the shape of the shadow price of financial resources. In this situation, the type of friction, financial on the one hand and technological or organizational on the other, cannot be identified from data on capital, output and the firm's value.

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1 Introduction

Research conducted in the 1990s has tried to remedy the empirical failure of the standard q theory of investment by introducing frictions in the investment process. One type of friction is financial: the firm's investment is constrained by its collaterizable resources. Thus, the firm faces liquidity constraints, which are motivated by asymmetric information between lenders and the firm (see Bernanke, Gertler & Gilchrist (1997) for a survey). The other type of friction is technological or organizational: investing means adjusting a capital stock, which may be costly. The shape of the underlying adjustment costs function is a source of irreversible, discontinuous or infrequent investment (see Caballero (1999) for a survey).

This note shows that in a simple model, namely a two-period deterministic framework, these two frictions can produce the same observed investment and the same present value of profits. Therefore, the type of friction, financial on the one hand and technological or organizational on the other, cannot be identified only from data on capital, output and the firm's value. It is shown that an adjustment cost function can imitate liquidity constrained investment, if the marginal cost of adjusting capital coincides with the shadow price of financial resources.

2 Liquidity Constraints

Consider a firm that maximizes profits over two periods in a deterministic environment without any external source of financing. Its only input is capital K and its internal financial position is the sum of profits $f(K)$ and undepreciated capital: $x(K) = f(K) + (1-\delta)K$, where f satisfies the Inada conditions, $f' > 0$, $f'' < 0$, and δ is the depreciation rate. Liquidity constraints are The firm's dividends, defined as profits net of investment I, are constrained to be nonnegative, that is, $f(K) - I \geq 0$. Capital accumulation satisfies the law of motion $K' = (1-\delta)K + I$, that is, capital next period equals undepreciated capital plus investment. The discount rate ρ is equal to the risk free interest rate. We can write the value function of the firm as

$$\begin{aligned} V(K) &= \max_{K'} \left\{ x(K) - K' + \frac{1}{1+\rho} x(K') \right\}, \\ \text{subject to} & \quad x(K) - K' \geq 0. \end{aligned}$$

This problem is solved with the Lagrangian $L(K', \xi) = (1+\xi)(x(K) - K') + \frac{1}{1+\rho} x(K')$. Setting $L_{K'} = -(1+\xi) + \frac{1}{1+\rho}(f'(K') + (1-\delta)) = 0$ and $L_\xi = 0$, the resulting solutions for capital and for the Lagrange multiplier, the shadow price of financial resources, are:

$$\begin{aligned} K^v(K) &= K'(K) = \min \left\{ x(K), f'^{-1}(\rho + \delta) \right\}, \\ \xi(K) &= \max \left(\frac{1}{1+\rho} (f'(x(K)) + (1-\delta)) - 1, 0 \right). \end{aligned}$$

If the firm's current capital is below its desired level, capital next period is increasing in current capital and the shadow price of financial resources is decreasing in the firm's financial resources (and in current capital). If the firm's current capital equals or exceeds its desired level, capital next period is set at its desired level and the shadow price of financial resources is zero. Having characterized this problem, we can inquire on the capital adjustment costs that yield this same policy rule.

3 Adjustment costs that mimic liquidity constraints

The problem is the same as in the previous section but with no constraint on dividends: now the firm can access external resources. However, when the firm changes its capital level from K to K', it has to incur in an adjustment cost that consists of a variable and a fixed component: $c(K', K) = g(K', K) + h(K)$. The value function of a firm operating in such an environment is

$$W(K) = \max_{K'} \left\{ x(K) - K' - c(K', K) + \frac{1}{1+\rho} x(K') \right\}$$

The Euler equation defines then the solution for this problem:

$$K^{w}(K)=\left\{K'\left|-1-c_1(K',K)+\frac{1}{1+\rho}\left(f'(K')+(1-\delta)\right)=0\right.\right\}.$$

Proposition 1 *The policy rule for capital with adjustment costs coincides with the policy rule with non-negative dividends,* $K^{w}(K)=K^{v}(K),\forall K,$ *if* $g_1(K^{v}(K),K)=\xi(K)$*, that is, the marginal cost of adjusting capital coincides with the shadow price of financial resources.*

Proof. An Euler equation that yields $K^{v}(K)$ must satisfy

$$1+c_1(K^{v},K)=\frac{1}{1+\rho}\left(f'(K^{v})+(1-\delta)\right)=1+\xi(K),$$

following that $c_1(K^{v}(K),K)=g_1(K^{v}(K),K)=\xi(K)$. ■

Corollary 2 *Proposition 1 implies that* $W(K)=V(K)-c(K^{v},K)$*. Therefore,* $W(K)=V(K),\forall K,$ *needs that* $c(K^{v},K)=g(K^{v}(K),K)+h(K)=0$*, that is,* $g(K^{v}(K),K)=-h(K)$*.*

Adjustment costs funcions that imitate liquidity constrained investment do not necessarily equate the two value functions V and W used to compute average q. To produce this additional result, adjustment costs at the optimum have to be zero, which implies negative fixed adjustment costs, or, in other words, that firms are compensated for the adjustment cost incurred.

We can inquire on functions that mimic liquidity constraints *and* conform to the usual shapes of adjustment cost functions like those used by Abel & Eberly (1994), Dixit & Pindyck (1993), and Cooper & Haltiwanger (2002). Let $I\equiv K'-(1-\delta)K$, then the variable component of adjustment costs has to satisfy:

$$g(K',K)\begin{cases}\geq 0 & if \quad I\neq 0,\\ =0 & if \quad I=0,\end{cases};g_1(K',K)\begin{cases}\geq 0 & if \quad I>0,\\ \leq 0 & if \quad I<0.\end{cases}$$

The authors mentioned above use 'general augmented adjustment cost functions' consisting of fixed, linear and quadratic components. Two examples of linear and quadratic components of adjustment cost functions that satisfy Proposition 1 are:

- Linear adjustment costs: $g(K', K) = \xi(K)(\chi - a_-(1-\chi)) I$,

  where $\chi = 1$ if $I > 0$ and $\chi = 0$ if $I < 0$. This function admits symmetry ($a_- = 1$), asymmetry and irreversibility ($a_- \neq 1$). The fixed cost that equates V and W is $h(K) = -\xi(K) f(K)$.

- Quadratic adjustment costs: $g(K', K) = \frac{\xi(K)}{2f(K)} I^2$.

  This function can also be extended to allow for asymmetries and irreversibilities. The fixed cost that equates V and W is $h(K) = -\frac{1}{2}\xi(K) f(K)$.

The common feature of these functions is that the 'scale effect', i.e., the effect of capital on adjustment, is the shadow price of financial resources $\xi(K)$, normalized by some function of profits.

4 Conclusion

In a two-period deterministic model several adjustment costs functions, including the popular linear and quadratic specifications, are able to mimic the investment pattern and present value of profits produced by a class of liquidity constraints, namely one characterized by non-negative dividends. This coincidence is attained when marginal adjustment costs equal the shadow price of financial resources. A matter of further research is extending this result to a stochastic environment with an infinite horizon and other types of liquidity constraints.

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