Economics Letters 63 (1999) 213–216

economics letters

Cost minimization and regulation in general equilibrium: an example

Sandro Brusco*

Universidad Carlos III de Madrid, Departamento de Economía de la Empresa, Calle Madrid 126, 28903 Getafe, Madrid, Spain

Received 26 January 1998; accepted 23 July 1998

Abstract

We present a general equilibrium example in which cost minimizing behavior prevents the economy from reaching the Pareto optimal allocation. In the example a firm with non-convex technology can use two different inputs to produce a single output. Pareto optimality requires that the firm use the more costly input, and it is therefore impossible to achieve the optimum when the firm minimizes costs.

Keywords: Non-convex technologies; General equilibrium; Regulation; Cost minimization

JEL classification: D51; L51

1. Introduction

The literature on the regulation of firms with increasing returns to scale (or, more generally, non-convex technologies) in a general equilibrium framework has focused on the existence and optimality of an equilibrium when some firms are forced by an external entity to follow pre-specified pricing rules (see e.g. Bonisseau and Cornet, 1988; Brown, 1991; Villar, 1994).

The extent of regulation considered in many papers belonging to this literature can be deemed excessive. In general, for example, the pricing rule imposed by the planner does not allow for cost minimization by the firm. An exception is Dierker et al. (1985). These authors propose a general equilibrium model in which non-convex firms are allowed to minimize their cost under the constraint that they satisfy demand, and the planner only dictates the prices of goods produced by non-convex firms. They prove the existence of an equilibrium under (among others) assumptions on the pricing rule followed by the planner and the technology of non-convex firms, but they do not consider the problem of optimal regulation.

When firms are not required to minimize cost, we know that Pareto optimality can be attained using marginal cost pricing. More exactly, we know that every Pareto optimum can be obtained as a

*Corresponding author. Tel.: +34-1-624-9307; fax: +34-1-624-9608.
E-mail address: brusco@emp.uc3m.es (S. Brusco)

0165-1765/99/$ – see front matter
PII: S0165-1765(99)00020-8

marginal cost pricing equilibrium (see Kahn and Vohra, 1987). The question is whether we can find a similar result when firms are allowed to minimize cost. Notice that we are not restricting attention to the marginal cost pricing rule. We want to know whether there is *any* pricing rule such that, when firms minimize costs, a Pareto optimum can be attained as an equilibrium.

This paper shows that the answer is no. We assume that the planner is only allowed to set prices for the output of the regulated firm and then prescribes that the firm is obliged to satisfy the demand existing at that price. However, the planner allows the regulated firm to select the input vector as it desires, and we will assume that inputs are selected to minimize the cost of producing the output. In this framework, we present an example in which cost-minimizing behavior is incompatible with Pareto efficiency, *for any pricing rule chosen by the regulator*. It is therefore impossible to attain a Pareto optimum unless the planner is allowed to interfere in the choice of inputs of the firm.

2. The example

There are three goods in the economy, x, y and z, and only one consumer with utility function $U(x, y, z) = xyz$. The consumer is endowed with ω_x units of good x and ω_z units of good z, and it will be assumed $2\omega_z > \omega_x \geq \omega_z > 0$. There is no endowment of good y. However, y can be produced through the following production function:

$$y^o = \max\{x^I, z^I\}$$

where x^I and z^I denote the quantity of x and z used as input, and y^o is the level of output. The Pareto optimum of this economy is computed solving the program

$$\max_{y^o, x^d, x^I, z^d, z^I} y^o x^d z^d$$

s.t.

$$y^o = \max\{x^I, z^I\}$$

$$x^d + x^I \leq \omega_x$$

$$z^d + z^I \leq \omega_z$$

The solution is $y^d = x^d = x^I = \omega_x/2$, $z^d = \omega_z$ and $z^I = 0$.

We will now show that there is no equilibrium price vector supporting this allocation when the firm minimizes cost. Assume that the firm production vector is $y^o = (\omega_x/2)$, $x^I = (\omega_x/2)$ and $z^I = 0$. Then the income of the consumer is:

$$Y = p_x\omega_x + p_z\omega_z + \left(p_y y^o - p_x x^I\right) = (p_x + p_y)\frac{\omega_x}{2} + p_z\omega_z$$

and demands are given by $y^d = (\frac{Y}{3p_y})$, $x^d = (\frac{Y}{3p_x})$ and $z^d = (\frac{Y}{3p_z})$. It is obvious that in the equilibrium supporting the Pareto optimal allocation all prices have to be positive, so that any good can be chosen as numéraire. Setting $p_y = 1$ we obtain the following equilibrium prices:

$$p_x = \frac{y^o}{\omega_x - x^I} = 1 \qquad p_z = \frac{y^o}{\omega_z - z^I} = \frac{\omega_x}{2\omega_z} \tag{1}$$

The important observation is that in this equilibrium we have $p_z < p_x$, but the firm uses input x, which is more expensive, rather than input z. Therefore the Pareto optimal allocation cannot be supported as an equilibrium when the regulated firm is allowed to minimize cost.

What level of production can be supported when the firm minimizes cost? First, it is easy to check that we have the (terribly inefficient) no-activity equilibrium:

$$p_y = 1 \quad p_z = p_x = 0 \;\; y^d = y^o = x^d = x^I = z^d = z^I = 0$$

The consumer has zero income, so she cannot buy the first unit of y. Without a positive amount of y, the goods x and z are worthless, so that $p_z = p_x = 0$ is justified. This is the same equilibrium that would obtain if no production possibility existed.

It is impossible to support equilibria in which $p_x > p_z$. This would imply $z^I = y^o$ and $x^I = 0$, which in turn implies $y^o/\omega_x > y^o/(\omega_z - y^o)$, or:

$$y^o < \omega_z - \omega_x \leq 0.$$

On the other hand, many levels of y^o can be supported when the input is x. This requires $p_x \leq p_z$ or $y^o/(\omega_x - y^o) \leq y^o/\omega_z$. Therefore, all levels of y^o satisfying $y^o \leq \omega_x - \omega_z$ can be supported as equilibria. However, since we have assumed $\omega_x < 2\omega_z$ it is clear that $y^o = \omega_x/2$ cannot be supported. We also observe that in the case $\omega_x = \omega_z$ the *only* equilibrium when the firm minimizes cost involves zero production. A planner trying to rule out this equilibrium imposing some pricing rule would cause a non-existence problem.

The impossibility of reaching the Pareto optimum is entirely due to the cost-minimizing behavior of the firm. As previously observed, we know from Kahn and Vohra (1987) that in the presence of non-convexities Pareto optima can still be reached as marginal cost pricing equilibria, which are (roughly) situations in which the vector of inputs and outputs chosen by the firm is such that the price is equal to marginal cost. This is the case in our example as well. The Pareto optimal allocation $y = x^I = x^d = \omega_x/2$, $z^d = \omega_z$, $z^I = 0$ is supported as a marginal cost pricing equilibrium by the prices computed in (1), that is:

$$p_y = 1 \quad p_x = \frac{y}{\omega_x - x^I} = 1 \quad p_z = \frac{y}{\omega_z - z^I} = \frac{\omega_x}{2\omega_z}$$

The firm is using x as input, so that marginal cost equals the price of output. However, the firm is not minimizing cost.

3. Comment

The basic ingredient of the example is that the firm is 'large' with respect to the economy, and its choice of inputs has a strong impact on the equilibrium price. In the example the firm has to choose between two alternative inputs, but whenever the ‘correct’ input is chosen its equilibrium price must be greater than the one of the input which is not chosen. The particular technology used in the example emphasizes this effect, since production efficiency requires the firm to use only one input. More in general, situations in which cost minimization by the regulated firm makes it impossible to achieve Pareto efficiency for any possible pricing rule appear to be likely in cases in which the choice of the input vector has a significant impact on the equilibrium price.

4. Conclusion

We have provided an example showing that in general equilibrium models with non-convex technologies it may be impossible to reach a Pareto optimal allocation if firms are allowed to minimize cost. While there are many examples in the general equilibrium literature in which inefficiencies appear when firms have non-convex technologies, such examples are usually linked to specified pricing rules. Our example shows instead that, in the presence of cost minimizing behavior, *every* pricing rule is bound to produce inefficiencies.

References

Bonisseau, J.M., Cornet, B., 1988. Existence of equilibria when firms follow bounded losses pricing rules. Journal of Mathematical Economics 17, 119–147.

Brown, D.J., 1991. Equilibrium analysis with non-convex technologies. In: Hildenbrand, W., Sonnenschein, H. (Eds.), Handbook of Mathematical Economics, Vol. IV, North-Holland, Amsterdam, Chapter 36.

Dierker, E., Guesnerie, R., Neuefeind, W., 1985. General equilibrium when some firms follow special pricing rules. Econometrica 53, 1369–1393.

Kahn, M.A., Vohra, R., 1987. An extension of the second welfare theorem to economies with non-convexities and public goods. Quarterly Journal of Economics 102, 223–241.

Villar, A., 1994. Equilibrium with non-convex technologies. Economic Theory 4, 629–638.