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HETEROGENEOUS DISTRIBUTION OF INFORMATION AND CONVERGENCE TO RATIONAL EXPECTATIONS EQUILIBRIUM IN A PARTIAL EQUILIBRIUM MODEL

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In models of convergence to rational expectations equilibrium it is usually assumed that information is (initially) concentrated upon a group of agents. We argue that this assumption is relevant for convergence results, and we show that when it is removed convergence may disappear.

1. Introduction

In recent years a number of papers have discussed the problem of convergence to the rational expectations equilibrium (henceforth, REE) when at least a subset of agents does not know the REE price function and has to learn it from the data. The usual assumption is that a subset of agents is perfectly informed about the realization of a random shock affecting agents' utilities, while other agents have to infer it. For a class of models it has been shown that, if agents use bounded rationality procedures such as OLS estimation to learn the 'true' price function, the economy will converge to the REE. In section 2 we present a model belonging to this class. In section 3 we show that the convergence result doesn't hold if, in the same model, we vary the initial distribution of information.

2. A partial equilibrium analysis

There are N agents in the market. They have equal characteristics, but are divided into two groups by the information owned. First group's members ('informed agents') know the realization of a random variable ϵ_t affecting their utility, while second group's members ('uninformed agents') do not. Let $\{\epsilon_t\}_{t=0}^{+\infty}$ be a sequence of random variables iid with support $(0, +\infty)$, and introduce the following assumptions:

Assumption 1. The demand of each informed agent at period t is

$$q_{it}^{\mathrm{d}} = (A/N) - (b/N)p_t + (c/N)\epsilon_t,$$

where p_t is the current price and A, b, c are positive parameters; the demand of each uninformed agent is

$$q_{it}^{\mathrm{d}} = (A/N) - (b/N)p_t + (c/N)F(\epsilon_t \mid \Omega_t),$$

where $F(\cdot \mid \Omega_t)$ is a forecasting rule given information Ω_t, and $\Omega_t = (\{p_\tau\}_{\tau=0}^{t}, \{\epsilon_\tau\}_{\tau=0}^{t-i})$.

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Assumption 2. The supply of the good is equal to A in each period. Let us suppose, without loss of generality, that informed agents are the first N_I. We put $\theta = N_I/N$. Given Assumptions 1 and 2 the equilibrium condition is

$$-bp_t + \theta c\epsilon_t + (1-\theta)cF(\epsilon_t \mid \Omega_t) = 0.$$

Since p_t is part of Ω_t, to compute the equilibrium price we have to specify how $F(\epsilon_t \mid \Omega_t)$ is formed.

In the REE all the information is revealed to every agent. Since informed agents know the realization of ϵ_t, in REE this information has to be known to uninformed agents. So, in REE, we have $F(\epsilon_t \mid \Omega_t) = \epsilon_t$. Substituting this expression of $F(\epsilon_t \mid \Omega_t)$ in the equilibrium condition we get the rational expectations price function, given by $p_t = (c/b)\epsilon_t$. Out of REE, we have to make assumptions on the behavior of agents having no a priori knowledge of the REE price function.

Assumption 3. $F(\epsilon_t \mid \Omega_t) = \hat{B}_{t-1} p_t$ with $\hat{B}_t = [\Sigma_{\tau-0}^{t} p_\tau \epsilon_\tau]/[\Sigma_{\tau-0}^{t} p_\tau^2]$.

Using Assumption 3 we can now compute the temporary equilibrium price in each period. We have

$$p_t = [\theta c\epsilon_t]/[b - (1-\theta)c\hat{\beta}_{t-1}]. \tag{1}$$

If we assume that the price can never be negative ('free disposal' assumption) then eq. (1) is constant and equal to zero for every $\hat{\beta}_{t-1} \geq [b/(1-\theta)c]$.

In every period the realization of p_t depends, trough $\hat{\beta}_{t-1}$ on every past event $\{\epsilon_\tau\}_{\tau-0}^{t-1}$ and on the expectation held at period zero. Anyway, the evolution of parameters can be represented as a stochastic difference equations system of the first order, defining variables $V_t - \Sigma_{\tau=0}^{t} p_\tau^2$ and $Z_t = \Sigma_{\tau-0}^{t} p_\tau \epsilon_\tau$, and considering the vector $H_t = (\hat{\beta}_t, p_t, V_t, Z_t)$. In each period, the evolution of H_t depends on the realization of ϵ_t and on the last period vector H_{t-1}. Convergence to REE obtains only if the stochastic process $\{H_t\}$ converges to a stochastic vector like $H^\circ = (b/c, c\epsilon_t/b, Z, bZ/c)$ where Z may take any positive value. Since we are only interested in the behavior of $\hat{\beta}_t$ and p_t we define the restricted vectors $h = (\hat{\beta}_t, p_t)$ and $h^\circ = (b/c, c\epsilon_t/b)$.

The first result we can establish is about stability of REE:

Proposition 1. If $h_{t-1} = h^\circ$ then $h_{t+j} = h^\circ$ for every $j \geq 0$.

The proposition is easily proved substituting $\hat{\beta}_{t-1} = b/c$ in eq. (1) and then substituting p_t in the definition of $\hat{\beta}_t$.

The REE is an 'absorbing point' of the stochastic process: once the economy has reached the REE it will remain at it for every realization of the sequence $\{\epsilon_\tau\}_{\tau-t}^{+\infty}$. But the REE is *not* the unique absorbing point. All points like $h = (\bar{\beta}, 0, V_\tau, Z_\tau)$ with $\bar{\beta} \geq (b/(1-\theta)c$ and $\tau = \{\min t \mid \hat{\beta}_t \geq b/(1-\theta)c\}$ are absorbing points. When $\hat{\beta}_t \geq b/(1-\theta)c$ the equilibrium price is zero; but if the equilibrium price is zero then $\hat{\beta}_{t+1} = \hat{\beta} t$, and this is true for all the following periods. This point needs some further comment. The stability of equilibrium hangs on the hypothesis we made about the agents' updating procedure. When economy reaches a REE it seems reasonable that agents keep their forecasting rule fixed, since forecastings are confirmed in every period. But when the economy reaches the no exchange equilibrium it seems rather unreasonable that agents keep fixed their forecasting rule. Agents can see that there is no relation between ϵ_t and the current price p_t and decide to change their forecasting rule. However we cannot say *how* the rule will be changed. To say something about this we have to introduce explicit hypotheses on the way agents tests their models and on the way they select new models. The question is beyond the scope of this work.

Let us consider now the evolution of $\hat{\beta}_t$. By mean of simple algebra, we can easily find that $\theta \geq \frac{1}{2}$ is a sufficient condition for $|\beta_t - (b/c)| < |\beta_{t-1} - (b/c)|$. The convergence result is now immediate.

Proposition 2. *If $\theta \geq \frac{1}{2}$ and $\beta_0 < b/[(1-\theta)c]$ then the estimate β_t converges to b/c.*

The proof involves the usual martingale argument, and can be requested to the author.

If the condition $\theta \geq \frac{1}{2}$ holds then many agents are certain about the environment. Therefore, the temporary equilibrium price is seriously affected by the realization of the random variable ϵ_t: the data are not 'too much' biased, and they can produce consistent estimates.

3. A generalization of the heterogeneous information case

In this section we investigate on the robustness of the convergence result to perturbations in the structure of initial information. We suppose that information is divided between two groups of agents, and we analyze the consequences of this when agents use a learning procedure similar to that of section 2. Let now $\epsilon_t = u_t + v_t$ with $\{u_t\}$, $\{v_t\}$ sequences of iid random variables with support $(0, +\infty)$ and $\text{cov}(v_t, u_\tau) = 0$ for every t, τ and assume that this is common knowledge.

Assumption 1′. The demand of first group's agents is

$$q_{it}^{\text{d}} = (A/N) - (b/N)p_t + (c/N) \cdot \left[u_t + F_1(v_t \mid \Omega_t^1)\right],$$

where

$$\Omega_t^1 = \left[\{u_\tau\}_{\tau-0}^{t}, \{v_\tau\}_{\tau-0}^{t-i}, \{p_\tau\}_{\tau-0}^{t}\right].$$

The demand of second group's agents is

$$q_{it}^{\text{d}} = (A/N) - (b/N)p_t + (c/N) \cdot \left[v_t + F_2(u_t \mid \Omega_t^2)\right],$$

where

$$\Omega_t^2 = \left[\{u_\tau\}_{\tau-0}^{t-i}, \{v_\tau\}_{\tau-0}^{t}, \{p_\tau\}_{\tau-0}^{t}\right].$$

Assumption 2′. $F_1(v_t \mid \Omega_t^1) = \hat{\beta}_{t-1} p_t$ with $\hat{\beta}_t = (\Sigma_{\tau-0}^t u_\tau \epsilon_\tau)/(\Sigma_{\tau-0}^t u_\tau p_\tau)$

$F_2(u_t \mid \Omega_t^2) = \tilde{\beta}_{t-1} p_t$ with $\tilde{\beta}_t = (\Sigma_{\tau-0}^t v_\tau \epsilon_\tau)/(\Sigma_{\tau-0}^t v_\tau p_\tau)$.

Given Assumptions 1′ and 2′, the temporary equilibrium price is

$$P_t = \left[\theta c u_t + (1-\theta) c v_t\right] \cdot \left[b - c\left(\theta \hat{\beta}_{t-1} + (1-\theta)\tilde{\beta}_{t-1}\right)\right]^{-1} \quad \text{for} \quad 0 \leq \theta\hat{\beta}_{t-1} + (1-\theta)\tilde{\beta}_{t-1} \leq b/c$$
$$= 0 \quad \text{otherwise.}$$

Temporary equilibrium price depends in a *distinct* way from the realizations of u_t and v_t: the coefficient of u_t is not equal to the coefficient of v_t, and the ratio is equal to $\theta/(1-\theta)$. The REE price function can be found impose $F_1(v_t \mid \Omega_t^1) = v_t$ and $F_2(u_t \mid \Omega_t^2) = u_t$; in this way we get $p_t = (c/b)\epsilon_t$, the same REE price function of section 2 (a natural consequence of the fact that information in the economy is the same). But things are very different about convergence. The following proposition gives us a very negative result.

Proposition 3. When forecasts are formed as in Assumption 2′, convergence to REE cannot obtain.

Proof. The temporary equilibrium price is

$$p_t = \theta c u_t / \left[b - c\left(\theta \hat{\beta}_{t-1} + (1-\theta)\tilde{\beta}_{t-1} \right) \right] + (1-\theta) c v_t / \left[b - c\left(\theta \hat{\beta}_{t-1} + (1-\theta)\tilde{\beta}_{t-1} \right) \right].$$

In the REE price function the random variables u_t and v_t must have the same coefficient, equal to c/b. This implies the following restrictions on coefficients:

$$\theta c \cdot \left[b - c\left(\theta \hat{\beta}_{t-1} + (1-\theta)\tilde{\beta}_{t-1} \right) \right]^{-1} = (c/b), \tag{1b}$$

$$(1-\theta) c \cdot \left[b - c\left(\theta \hat{\beta}_{t-1} + (1-\theta)\tilde{\beta}_{t-1} \right) \right]^{-1} = (c/b). \tag{2}$$

But this conditions may hold only if $\theta = 1 - \theta$, i.e., $\theta = \frac{1}{2}$. So, when $\theta \neq \frac{1}{2}$, there are no values of $\hat{\beta}_t$ and $\tilde{\beta}_t$ making a REE possible. Consider now the $\theta = \frac{1}{2}$ case. Imposing the additional restriction $\hat{\beta}_t = \tilde{\beta}_t$ we get $\hat{\beta}_t = \tilde{\beta}_t = (b/2c)$. If, at period $t-1$, this condition is satisfied, then at period t we have $p_t = (c/b)\epsilon_t$ and the new observation (p_t, u_t, v_t) leads to the new estimates

$$\hat{\beta}_t = (b/c) - (b/2c) \cdot \left[1 + (c/b) u_t \epsilon_t / \Sigma_{\tau=0}^{t-i} u_\tau p_\tau \right]^{-1},$$

$$\tilde{\beta}_t = (b/c) - (b/2c) \cdot \left[1 + (c/b) v_t \epsilon_t / \Sigma_{\tau=0}^{t-i} v_\tau p_\tau \right]^{-1}.$$

For u_t and v_t assume only strictly positive values; we have

$$\hat{\beta}_t > (b/2c); \; \tilde{\beta}_t > (b/2c); \; \hat{\beta}_t + \tilde{\beta}_t > b/c.$$

So, we get out from the REE point in only one period for every realization of u_t and v_t. Q.E.D.

When information's distribution is like that of Assumption 1′, no agent knows the realization of ϵ_t. So, unless we are in REE, no agent acts in certainty conditions and the value of p_t is not 'sufficiently well' linked to ϵ_t: the data on which estimates are carried on are not 'good enough' and convergence does not obtain. Proposition 3 shows that, when we look at convergence in regime of bounded rationality, the usual dichotomic structure of information's initial distribution is not a good rappresentation of the general case. We have shown an example where the information's initial distribution is not relevant for the REE existence problems, but the 'learning by OLS estimation' procedure, often used in the literature, yields very different results when information is spread over two groups.

References

Blume, L. and D. Easley, 1982, Learning to be rational, Journal of Economic Theory 26, 340–351.

Bray, M.M., 1982, Learning, estimation and the stability of rational expectations, Journal of Economic Theory 26, 318–339.

Bray, M.M. and D. Kreps, 1981, Rational learning and rational expectations, Research paper no. 616 (Graduate School of Business, Stanford University, Stanford, CA).

Bray, M.M and N.E. Savin, 1986, Rational expectations equilibria, learning and model specification, Econometrica 54, 1129–1160.

De Canio, S.J., 1979, Rational expectations and learning from experience, Quarterly Journal of Economics 92, 47–57.

Margaritis, D., 1987, Strong convergence of least squares learning to rational expectations, Economics Letters 23, 157–161.