

Without the Born Rule

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The auxiliary rules of quantum mechanics have always included the “Born rule” that connects probability with square modulus. This need not be the case, for it is possible to introduce probability into the theory through probability current alone. When this is done, other rules can provide for stochastically triggered measurements within a system of any size, microscopic or macroscopic; and solutions to the Schrödinger equation can be consistently applied to individual trials, not just to ensembles of trials. Other advantages appear. The rules can then resolve the paradox associated with the Schrödinger cat experiment, and remove the possibility of the many world thesis of Everett. As a result, the system can accommodate any conscious observer, including the principal investigator who cannot otherwise be included in a quantum mechanical system.

Introduction

Schrödinger’s equation is always accompanied by a set of auxiliary rules that say how it is to be used. The Copenhagen school gives one set of rules, four of which are listed in another paper [1]. They include the Born rule, instructions relating to the use of primary data, and the collapse of the wave function, etc. Other rule-sets of standard quantum mechanics have been proposed in the past 80 years, some of which deny that there is a collapse of the wave function. However, they all have one thing in common. They all include the Born rule that functions as the sole connection between theory and observation. I will call any one of these rule-sets the *sRules*, indicating that they are auxiliary rules that accompany standard quantum mechanics.

The Born rule is not necessary. It is possible to achieve very desirable results by dropping the Born rule from the governing auxiliary rules and introducing probability into quantum mechanics through *probability current* alone. This has been done with two very different auxiliary rule-sets called the *nRules* and the *oRules*. In this paper we are only concerned with the nRules.

The nRules

For the most part, solutions to Schrödinger's equation represent a continuous and classical-like evolution of a quantum mechanical wave function. Occasionally this evolution engages a state that is discontinuous with its immediate predecessor in some variable. This may be called a quantum *jump* or quantum *gap*. The nRules are concerned with what happens at one of these gaps.

We choose the nRules with two objectives in mind. First, the rules should locate any measurement inside of the system being measured, and should apply equally to all systems independent of size – microscopic or macroscopic. A measurement is understood to mean a stochastically initiated collapse of the wave that occurs independent of an 'external' measuring device or observer. Second, the rules should apply to individual trials, not just to ensembles of trials. When these objectives are met, we find that the system is able to include conscious observers without the ambiguity associated with the Schrödinger cat experiment, or with the many worlds of Everett.

The first of these rules recognizes two different kinds of states: *ready states* and *realized states*. A ready state \underline{S} is underlined, and a realized state S is not. The first rule tells us that ready states are introduced into solutions of Schrödinger's equation immediately after a quantum jump. A state is otherwise realized. We define a *ready component* as one that includes one or more ready states. It too exists immediately after a quantum jump.

nRule (1): *If an irreversible interaction produces a complete component that is discontinuous with its predecessor in some variable, then it is a ready component.*

Complete components contain all of the symmetrized objects in the universe. Each included object is itself complete in that it is not just a partial expansion in some representation. The ready states in a discontinuous component are the discontinuous states appearing there.

The second rule establishes the existence of a stochastic trigger that stochastically chooses ready components. This is a property of the system that has nothing to do with the kind of interaction taking place or its representation. The trigger is independent of representation because it is the entire (complete) component that is stochastically chosen.

The flow per unit time of square modulus between components is given by the modular current J , and the total square modulus of the system is given by s . The square modulus is not here identified with probability; rather, probability is introduced only through *probability current* J/s . The division of J by s automatically normalizes the system at each moment of time. So currents rather than functions are normalized in this treatment.

nRule (2): *A systemic stochastic trigger strikes a ready component with a probability per unit time equal to the positive probability current J/s flowing into it.*

The collapse of a wave function and the change of a ready state to a realized state is given in nRule (3).

nRule (3): *When a ready component is stochastically chosen, all the included ready states become realized states, and all other components go to zero.*

A complete statement of the first three rules is given in Appendix A together with other relevant definitions and needed clarifications. There is a fourth nRule. However, we will illustrate the first three rules before going on.

Particle and Detector Interaction

We apply Schrödinger's equation to a 'microscopic' particle interacting with a 'macroscopic' detector in order to demonstrate the first three nRules. These two objects are assumed to be initially independent and given by the equation

$$\Phi(t) = \exp(-iHt)\psi_i \otimes d_i \quad (1)$$

where ψ_i is the initial particle state and d_i is the initial detector state. The particle is then allowed to pass over the detector, where the two interact with a cross section that may or may not result in a capture. After the interaction begins at a time t_0 , the state is an entanglement in which the particle variables and the detector variables are inseparable. In general, a direct product of states such as ψd (in Eq. 2 below) indicates an interaction that, in most cases, leads to or has led to an entanglement.

The first component in Eq. 2 includes the detector d_0 in its ground state prior to capture. The second component \underline{d}_1 is the detector in its capture mode.

$$\Phi(t \geq t_0) = \psi(t)d_0(t) + \underline{d}_1(t) \quad (2)$$

where the capture state is zero at t_0 and increases in time. It is a 'ready' state as prescribed by nRule (1), for the particle goes discontinuously and irreversibly from being spread throughout space in the first component to being inside the detector in the second component – like going discontinuously from the 1st to the 2nd orbit of an atom.

The state $\psi(t)$ is an incoming free particle plus all of the scattered components that are correlated with recoil states of the ground state detector. The second component is a superposition that includes all of the recoil components of the detector that have captured the particle¹.

As probability current flows into the second component in Eq. 2, it may take a stochastic hit at some time t_{sc} following nRule (2). In that case the state will collapse following nRule (3) to yield

$$\Phi(t \geq t_{sc} > t_0) = d_1(t)$$

¹ Each component in Eq. 2 has an attached environmental E_0 and E_1 that is not shown. These are orthogonal to one another, insuring local decoherence. But even though Eq. 2 may be decoherent locally, we assume that the macroscopic states d_0 and d_1 are fully coherent when E_0 and E_1 are included. So Eq. 2 and others like it in this paper are understood to be coherent when universally considered. We call them "superpositions", reflecting their global rather than their local properties. Superpositions of mesoscopic states have been found at low temperature [2]. The components of these states are locally coherent for a measurable period of time.

where d_1 becomes a realized state and the first component disappears. Renormalization is not necessary because the current flow into $\underline{d}_1(t)$ is continuously normalized according to nRule (2). It is explained in Ref. 1 how to treat this case when there is no capture of the particle.

So far there are no difficulties with these nRules; however, difficulties arise when there are three or more components as in the case of a particle counter.

A Counter

If a particle counter is exposed to a radioactive source, its state function would normally be written

$$\Phi(t \geq t_0) = C_0(t) + C_1(t) + C_2(t) + C_3(t) + \text{etc.} \quad (3)$$

where C_0 is the initial state of the counter (registering 0 counts) at time t_0 . The state C_1 registers 1 count, C_2 registers 2 counts, etc. The radioactive source and the particle field are not shown in Eq. 3. The components following C_0 are zero at t_0 ; and after that, they become non-zero in the form of a pulse that travels from left to right in Eq. 3.

Here is the difficulty. We want the nRules to apply to macroscopic systems as well as microscopic systems. If the interaction is a long-lived radioactive decay, then the pulse in Eq. 3 might be spread over two or more components at the same time. It might overlap both C_1 and C_2 , causing probability current to flow simultaneously into each one. This means that there might be a stochastic hit on C_2 *before* there is a hit on C_1 , and that would cause a collapse of the state that goes directly to C_2 . The state C_1 would then be skipped over. This is a very unphysical result for a macroscopic body, and the fourth nRule is designed to insure that that does not happen.

The Fourth nRule

Only positive current going into a ready component is physically meaningful because it represents positive probability. A negative current (coming out of a ready component) is not physically meaningful and is not allowed by nRule (4).

nRule (4): *A ready component cannot transmit probability current to other components or advance its own evolution.*

Although it can receive current that increases its square modulus, a ready state is dynamically terminal. It cannot develop beyond itself or contribute to the development of anything else. It is shown in the Appendix B that this property is arranged by modifying the Hamiltonian of the system. When that is done the first term $C_0(t)$ in Eq. 3 decreases in time as the second component $C_1(t)$ increases in time, conserving square modulus; but $C_1(t)$ no longer evolves dynamically beyond the values given it by $C_0(t)$ at each moment of time. The correct nRule equation is therefore

$$\Phi(t \geq t_0) = C_0(t) + \underline{C}_1(t) \quad (4)$$

replacing Eq. 3. This insures that \underline{C}_1 will be chosen. It will not be skipped over.

There is no general theory that determines the Hamiltonian associated with Eq. 4. In standard theory it is found from long experience to take the form $H_s = H_0 + H_{01} + H_1$, where H_0 and H_1 drive the first and second components respectively, and H_{01} drives the interaction between them. In the end, the justification for this form is that it works. However, nRule (4) forces a change. The Hamiltonian now takes the truncated form $H = H_0 + H_{01}$, where the last term H_1 has been dropped (see Appendix B). This too works, but it works differently. It works when combined with the other nRules. The effect of this change is demonstrated in the remainder of this paper.

Think of $\underline{C}_1(t)$ in Eq. 4 as the *launch component* of a next solution of Schrödinger's equation if and when it is stochastically chosen. It contains all the boundary conditions of that solution. Those boundary conditions are carried over from C_0 directly into \underline{C}_1 at each moment of time (through H_{01}), but they are not applied to the next solution of the Schrödinger equation until the moment of collapse to that solution. If there is a stochastic hit on \underline{C}_1 at time t_{sc} , then nRule (3) will require a collapse to the next solution that is the realized state C_1 .

$$\Phi(t \geq t_{sc} > t_0) = C_1(t) + \underline{C}_2(t) \quad \text{now using } H_1 + H_{12} \quad (5)$$

where the ready state $\underline{C}_2(t)$ is the launch component into the 'next' solution of the Schrödinger equation.

We see that the *single* equation given by the sRules of standard quantum mechanics in Eq. 3 is replaced by a separate equation for each quantum jump, plus one. So the first quantum jump in Eq. 3 is replaced by *two* equations under the nRules – one *before* the stochastic hit (Eq. 4), and one *after* the stochastic hit (Eq. 5). This is characteristic of the difference between the sRules and the nRules. Whereas the sRules allow only one solution beginning with the initial conditions, the nRules take on new boundary conditions and hence new solutions to the Schrödinger equation after each stochastic hit. The sRules run all these solutions together as though they all occur at the same time. The nRules take them one at a time in their proper sequence.

Since the nRules are intended to govern microscopic as well as macroscopic systems, equations like 4 and 5 should apply as well to atomic states. If the atom goes through a series of decays from levels $A_5(t)$ to $A_2(t)$ and then to the ground level $A_0(t)$, it will do so in separate steps.

$$\Phi(t \geq t_0) = A_5(t) + \underline{A}_2(t) \quad \text{using } H_5 + H_{52} \quad (6)$$

followed by $\Phi(t \geq t_{sc1} > t_0) = A_2(t) + \underline{A}_0(t)$ using $H_2 + H_{20}$

and then $\Phi(t \geq t_{sc2} > t_{sc1} > t_0) = A_0(t)$ using H_0

where t_{sc1} and t_{sc2} are the times of the two stochastic hits that initiate the two decays. Starting with the initial conditions $A_5(t_0)$, the first solution in Eq. 6 prepares the launch state $\underline{A}_2(t)$ of the second solution which contains all of the required boundary conditions of $A_2(t)$ at the time of the first decay. The second solution prepares the launch state $\underline{A}_0(t)$ of the final solution which contains all of the required boundary conditions of $A_0(t)$ at the time of the second decay. In each case the launch component is not “launched” until it is stochastically chosen, for a ready component cannot evolve dynamically on its own.

On the other hand, the sRules will run these solutions together into one equation. Experimentally, the ensemble of photons of either frequency that is received during any interval of time after t_0 is the same for the nRules as is for the sRules.

Parallel Branching

Now imagine parallel states in which a quantum process may go either clockwise or counterclockwise as shown in Fig. 1. Each component includes a macroscopic piece

of laboratory apparatus A , where the Hamiltonian provides for an irreversible and discontinuous clockwise interaction going from the 0^{th} to the r^{th} state and another one from there to the final state f ; as well as a comparable counterclockwise interaction from the 0^{th} to the l^{th} state and from there to the final state f . The Hamiltonian does not provide a direct route from the 0^{th} to the final state, so the system chooses stochastically between a clockwise and a counterclockwise route. Launch states \underline{A}_l and \underline{A}_r are the eigenstates of that choice, and contain the boundary conditions of each choice.

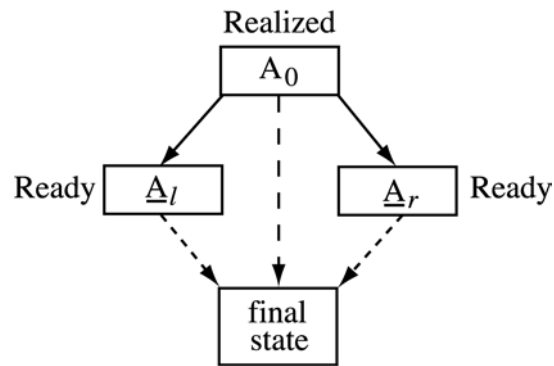


Figure 1: Possible parallel decay routes

With nRule (4) in place, probability current cannot initially flow from either one of the intermediate states to the final state, for that would require current flow out of a ready state. The dashed lines in Fig. 1 indicate the forbidden transitions. But once the state \underline{A}_l (or \underline{A}_r) has been stochastically chosen, it will become a realized state A_l (or A_r) and the final state will become \underline{A}_f . A subsequent transition to \underline{A}_f can occur that realizes A_f .

The effect of nRule (4) is therefore to force this macroscopic system into a classical sequence that goes either clockwise or counterclockwise. Without it, the system might make a second order transition (through one of the intermediate states) to the final state without the intermediate state being realized. The observer might then see the initial state followed by the final state, without knowing (in principle) which pathway was followed. This is a property of quantum mechanical systems that applies to continuous microscopic processes under the nRules as well as the sRules. In Heisenberg's famous example formalized by Feynman, a microscopic particle observed at point a and later at

point b will travel over a quantum mechanical superposition of all possible paths in between. Without nRule (4), macroscopic objects facing discontinuous and irreversible parallel choices would do the same thing. But that should not occur. The fourth nRule forces this parallel system into one or the other path, so it is not a quantum mechanical superposition of both paths.

The same would be true of a microscopic parallel system of discontinuous paths. Suppose A_0 is an initial atomic energy level where there are two separate paths to the ground state A_f . The sRules say that these two paths are in superposition. Everett's many-world theory furthermore tells us that the two paths will not be further involved with one another – except in this case, since as they arrive at the same final state. The nRules tell a different story. They deny that these two paths are in superposition. They have no intermediate involvement with one another other *because* only one of them happens at a time. It's a classical either/or situation, even at this atomic level.

Free Neutron Decay

Another interesting example is that of a free neutron decay. This is written

$$\Phi(t > t_0) = n(t) + \underline{ep\bar{v}}(t)$$

where the second component is zero at $t = 0$ and increases in time as probability current flows into it. This component contains three entangled particles that make a whole object, where the three together are a single ready component as indicated by the underline (see nRule 1). Following nRules (2) and (3), there will be a stochastic hit on $\underline{ep\bar{v}}(t)$ at some time t_{sc} , reducing the system to the realized correlated states $\underline{ep\bar{v}}(t_{sc})$.

This case provides a good example of how $\underline{ep\bar{v}}(t)$ is a function of time beyond its increase in square modulus. Assume that the neutron moves across the laboratory in a wave packet of finite width. At each moment the ready component $\underline{ep\bar{v}}(t)$ will ride along with the packet, with the same size, shape, and group velocity. It is the launch component that contains the boundary conditions of the next solution of the Schrödinger equation – the one that appears when $\underline{ep\bar{v}}(t)$ is stochastically chosen at time t_{sc} . Before this 'collapse', $\underline{ep\bar{v}}(t)$ is time dependent because it increases in square modulus *and* because it follows the motion of the neutron. However, nRule (4) insures that it will not

evolve dynamically beyond itself until it becomes a realized component at the time of stochastic choice. The neutron $n(t)$ will then disappear and the separate particles in the launched component $ep\bar{v}(t)$ will spread out on their own, although correlated in conserved quantities.

Specific values of, say, the electron's momentum are not stochastically chosen by this reduction because all possible values of momentum for each particle are included in $ep\bar{v}(t)$ and its subsequent evolution (after ΔE is resolved). For the electron's momentum to be determined in a specific direction away from the decay site, a detector in that direction must be activated. That will require another stochastic hit on, and collapse to, the component that includes that detector.

Spin Correlated Fermions

Still another example is a pair of correlated fermions $f_1 f_2$, where the direct product generally assumes an interaction that leads to, or has led to, an entanglement of some kind. It is not necessary to be specific about this entanglement in order to write the minimal equation (under the nRules) of a measurement of the spin of the second particle. This is given by

$$\Phi(t \geq t_0) = f_1 f_2 \underline{M} + E_1(\uparrow) \underline{M}(\downarrow) + E_1(\downarrow) \underline{M}(\uparrow)$$

where the measuring device $\underline{M}(\downarrow)$ now includes the second fermion and records a spin down, and the measuring device $\underline{M}(\uparrow)$ includes the second fermion and records a spin up. In general, the FM products might also be entanglements. Current flows from the first component to both of these launch components. If there is a stochastic hit on the second component at time t_{sc} we will have

$$\Phi(t \geq t_{sc} > t_0) = F_1(\uparrow) \underline{M}(\downarrow)$$

Add an Observer

Assume that an observer is looking at the detector in Eq. 1 from the beginning.

$$\Phi(t) = \exp(-iHt) \psi_i \otimes D_i B_i \quad (7)$$

where B_i is the observer's initial brain state that interacts with the detector D_i . The brain is understood to include only higher order brain parts – that is, the physiology of the brain

that is directly associated with consciousness after all image processing is complete. All lower order physiology leading to B_1 is assumed to be part of the detector. The detector is now represented by a capital D indicating that it includes the bare detector plus the low-level physiology of the observer.

Following the interaction between the particle and the detector, Eq. 2 becomes

$$\Phi(t \geq t_0) = \psi(t)D_0(t)B_0 + \underline{D}_{1w}(t)B_0 \quad (8)$$

where the launch component is zero at t_0 and increases in time. The state B_0 in the first component is the observer's conscious brain state that interacts with the detector $D_0(t)$, and is therefore aware of its 'zero' reading. The launch state $\underline{D}_{1w}(t)$ represents the capture state of the detector when only its window end had been affected. When the particle enters the window of the detector it initiates an irreversible process that defines a ready component, and then it freezes because of nRule (4). This means that except for the particle's presence just inside the window, the launch component in Eq. 8 has the *same content* as the first component. Therefore, B_0 is the brain state that appears in the launch component. Physically, this reflects the fact that the observer at the display end will not experience the capture until its effect has moved classically through the detector to the display, and that does not happen until *after* a stochastic hit on $\underline{D}_{1w}(t)B_0$.

With that hit on $\underline{D}_{1w}(t)B_0$ at time t_{sc} , the system becomes

$$\Phi(t \geq t_{sc} > t_0) = D_{1w}(t)B_0 \rightarrow D_{1d}(t)B_1 \quad (9)$$

where the realized component $D_{1w}(t)B_0$ evolves continuously and classically (represented by the arrow) until it reaches the display end of the detector (represented by D_{1d}). At this point the brain state B_1 of the observer becomes aware of the capture. The change in the conscious brain state from B_0 to B_1 in Eq. 9 is continuous and classical.

Compare this with an sRule evaluation of this capture. In that case Eq. 8 would be given by

$$\Phi(t \geq t_0) = \psi(t)D_0(t)B_0 + D_{1w}(t)B_0 \rightarrow D_{1d}(t)B_1 \quad (\text{standard QM})$$

where only $\psi(t)D_0(t)B_0$ is initially non-zero. The plus sign again refers to a discontinuous quantum jump and the arrow refers to a continuous classical evolution. So as probability current builds up the second (window) component, current will begin to flow from the window to the display state in a continuous evolution. If the particle in this

equation is produced by a long-lived radioactive decay, this single equation might very well contain both B_0 and B_1 in superposition. The observer would then be in two different states of consciousness *at the same time*. Here we have another formulation of Schrödinger's famous cat paradox, which is an unacceptable ambiguity.

The nRules tell a different story. According to the nRules there are two different equations in Eqs. 8 and 9. One describes the system *before* the stochastic hit, and the other describes the system *after* the stochastic hit. Furthermore, there is only one brain state in Eq. 8, and there is only one brain state 'at a time' in Eq. 9, so there is *no* paradoxical cat-like ambiguity. Again, the sRules run both solutions together as though they both occur at the same time, giving rise to the ambiguity; whereas the nRules provide separate solutions that apply before and after the stochastic hit, eliminating the ambiguity. Each of these separate solutions has its own boundary conditions. The initial conditions apply to Eq. 8, and the boundary conditions created by the launch state in Eq. 8 apply to Eq. 9.

Many Parallel Sequences

Imagine three counters that surround a single radioactive source. The system state A_0 means that none of the counters have yet recorded a count. The system state A_1 means the first counter is the first to record a count, A_2 means that the second counter is the first to record a count, etc. Each of the three counters is exposed to two other radioactive sources a and b that will become active after the first choice has been made. The system state A_{xa} (or A_{xb}) indicates which of these two sources produces a second count of the A_x counter. An observer is on hand to determine which of the six possible sequences is realized in an individual trial. These sequences are shown in Fig. 2, where AB_0 means that the brain B observes the system in the state A_0 , etc. In writing the

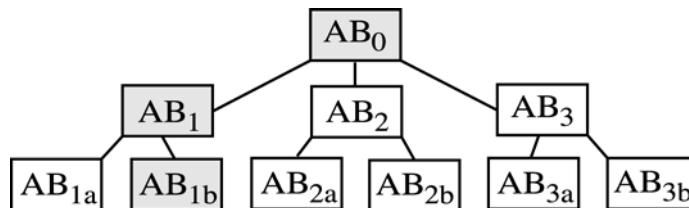


Figure 2: Six possible sequences

component this way, we overlook the detail associated with the classical evolution that occurs inside the apparatus. We take only the end result of the observer interacting with the apparatus. The shaded sequence in Fig. 2 is the one discussed below.

The sRules run all these possibilities together in a single superposition. This makes sense if the superposition refers only to an ensemble. But the theory would not then apply to individual trials, and this is a theoretical limitation we want to avoid. On the other hand, if the superposition is applied to individual trials, the result is again paradoxical. It means that the single observer would experience all possible sequences at once. This is the *many world* theses of Everett, according to which the observer in Fig. 2 splits up into six separate conscious entities. Everett shows that this does not lead to a contradictory confrontation between any two of the alter-egos because any sequence, once begun, will proceed without any further involvement with any other sequence [3].

The nRules resolve this ambiguity. After t_0 and before there has been a hit on any counter, the system is given by

$$\Phi(t \geq t_0) = A_0(t)B_0 + \underline{A}_1(t)B_1 + \underline{A}_2(t)B_2 + \underline{A}_3(t)B_3 \quad (10)$$

where only $A_0(t)B_0$ is initially non-zero. This is a parallel system in which current flows to all three eigen-components $\underline{A}_1(t)B_1$, $\underline{A}_2(t)B_2$, and $\underline{A}_3(t)B_3$ that are the possible launch states of the next solution².

After the first stochastic hit at time t_{sc1} , imagine that the chosen reduced state is

$$\Phi(t \geq t_{sc1} > t_0) = A_1(t)B_1 + \underline{A}_{1a}(t)B_{1a} + \underline{A}_{1b}(t)B_{1b}$$

where only $A_1(t)B_1$ is initially non-zero. The realized component in this equation is equal to the second component in the shaded sequence in Fig. 2. Again, we overlook the classical evolution that occurs inside the counter in this equation, so B_x is shown together with A_x .

A second hit gives the final state

$$\Phi(t \geq t_{sc2} > t_{sc1} < t_0) = A_{1b}(t)B_{1b}$$

where the realized component in this case is the third shaded component in Fig. 2, and where again, the classical evolution inside the detector is overlooked.

² The brain state in the launch components of Eq. 10 should be ready states according to nRule (1). But if the internal classical evolution is considered, ready brains states would not exist. The notation in Eq. 10 reflects the latter consideration.

The nRules do not support a superposition of all the sequential possibilities. Instead, they produce only one sequence in a single trial that reflects the stochastic choices that have been made in that trial. So the nRules apply to individual trials *and* they avoid the unphysical many-world interpretation of Everett. The different sequences, once begun, have no further involvement with one another *because* they occur separately as classical either/or choices.

Other Cases

In Ref. 1, the nRules are applied to a number of other cases with good results. It is shown that the Born rule follows as a theorem when the nRules are applied to a terminal observation in a typical physics experiment. The nRules also work well when the observer does not initially interact with a detector (as in Eq. 7), but comes on board *during* the primary interaction between the particle and the detector. The nRules also work as they should when a second observer is brought on board to share an observation with the first observer.

Also in Ref. 1, several other microscopic systems are analyzed using the nRules. We look at atomic emission and absorption, separating out the influence of spontaneous vs. stimulated emission. Also, Rabi oscillations and decoherence are investigated. It is shown that both, by themselves, avoid creating ready states and stochastic reduction. We also examine the effect of magnetic fields on spin states. It is shown that there are no stochastic reductions prior to the spin system interacting with a ‘location’ detector after it has being deflected by the field.

In another paper [4], the nRules are applied the Schrödinger cat experiment in its various forms. In the first version of that experiment, the cat is initially conscious and is rendered unconscious as a result of a beta decay. In the second version, the cat is initially unconscious and is awoken by an alarm that is stochastically triggered by a beta decay. In a final version, the cat initially unconscious and is awakened by an internal alarm that is in competition with the above external alarm. In all these cases, the nRules successful predict the expected experience of the cat. In addition, the nRules successfully predict

the experience of an external observer who opens the box (containing the cat) at any time during any one of the above experiments.

Observers Included

The Born rule is not only one of the sRules, but it is taken to be the sole connection between the theory and observation. That connection is an instantaneous contact. The Born rule observer can only ‘peek’ at the system from time to time, approaching a continuous experience only in a Zeno-like approximation. The primary observer cannot himself be a continuous part of the system under the sRules. Furthermore, while the sRules allow a secondary observer (such as Schrödinger’s cat) into the system in a continuous way, they predict that that observer can be in two different states of consciousness at the same time. I believe that to be a false prediction rather than an occasion to speculate about the existence of multiple alter-egos. Physics should construct theory to accommodate observable phenomena – not invent ‘unobservable’ phenomena to accommodate theory.

On the other hand, the nRules make a place for the primary and the secondary observer. The brain states referred to in the above were presumably those of a secondary observer. There was never any ambiguity about what those brains experienced at any moment of time in any one of the separate solutions; so quantum theory with these auxiliary rules accurately predicts the experiences of any included observer. This *is* the connection between theory and observation under the nRules. It is the same as that theory/observation connection in classical physics. Also as in classical physics, the primary observer who is investigating a system can always look at a wider system that includes himself in a continuous way. A model of his own brain is then included in the system, correctly predicting his experience there.

The oRules

There is another rule-set that rejects the Born rule and relies on probability current to introduce probability into quantum mechanics [5, 6]. These are called the *oRules*. They differ from the nRules in that only brain states can be ready states. So only brains

can be the basis states of a state reduction. These rules reflect the notions of Wigner [7] and von Neumann [8] and others to the effect that the presence of a conscious observer is necessary to a state reduction. These rules seem to be every bit as adequate as the nRules, and I do not make a final choice between the two. In the end we do not really know what is required for the collapse of a wave function, so I believe that all of the consistent and observationally accurate auxiliary-rule options should be on the table.

Appendix A

nRule (1): *If an irreversible interaction produces a complete component that is discontinuous with the initial component of a solution, then all of the new states that appear in this component will be ready states. All other states will be realized.*

[**note:** A *complete component* is a solution of Schrödinger's equation that includes all of the (symmetrized) objects in the universe. It is made up of complete states of those objects, including all their state variables. A component that is a sum of less than the full range of a variable (such as a partial Fourier expansion) is not complete.]

[**note:** Continuous means continuous in all variables. Although solutions to Schrödinger's equation change continuously in time, they can be *discontinuous* in other variables - e.g., the separation between the n th and the $(n + 1)^{\text{th}}$ orbit of an atom with no orbits in between. A discontinuity can also exist between macroscopic states that are locally decoherent. For instance, the detector state d_0 (the ground state) and d_1 (the capture state) are discontinuous with respect to detector variables. There is no state in between. Like atomic orbits, these detector states are considered to be 'quantum jump' apart. The continuous 'internal' evolution of a detector that follows a jump is not part of the jump. The jump only occurs at the *window* end of a detector, and is complete as soon as something irreversible happens (inside the detector) to secure the measurement.]

[**note:** The *initial component* is the first complete component that appears in a given solution of Schrödinger's equation. A solution is defined by a specific set of boundary conditions. Boundary conditions change with the collapse of the wave function.

The single component that survives a collapse will be complete, and will be the initial component of the new solution.]

nRule (2): *For a system of total square modulus s that has n ready components, a stochastic trigger will choose stochastically from among them. The probability per unit time of a choice among m of these components at time t is given by $(\sum_m J_m)/s$, where the square modular current J_m flowing into the m^{th} component at that time is positive.*

[**note:** Dividing J by s insures normalization at every moment of time. There is no need to normalize after each state reduction because currents, not functions, are normalized in this treatment.]

nRule (3): *If a ready component is stochastically chosen, then all of the included ready states will become realized, and all other components in the superposition will be immediately reduced to zero.*

[**note:** The claim of an immediate (i.e., discontinuous) reduction is the simplest way to describe the collapse of the state function. A collapse is brought about by an instantaneous change in the boundary conditions of the Schrödinger equation, rather than by the introduction of a new 'continuous' mechanism of some kind.]

[**note:** This collapse does not preserve normalization. That does not alter the probability of subsequent reductions because of the way probability per unit time is defined in nRule (1); that is, current J is divided by the total square modulus. Again, Currents are normalized in this treatment, not functions.]

Appendix B

Assume that a unitary operator $U(t)$ displaces the system in time. An instantaneous translation through time is then given by

$$U(dt) = 1 - iHdt$$

or

$$U(t + dt) = U(dt) U(t) = (1 - iHdt)U(t)$$

where H is a Hermitian Hamiltonian. Applying it to $\Phi(t)$ gives

$$\Phi(t + dt) = (1 - iHdt)\Phi(t) = \Phi(t) - iH\Phi(t)dt \quad (\text{B1})$$

from which we get Schrödinger's equation

$$H\Phi(t) = (id/dt)\Phi(t)$$

Conservation of probability current follows from this. Also, operators that commute with this Hamiltonian are constants of the motion.

We then choose the Hamiltonian that drives the system

$$\Phi(t) = S_1(t) + \underline{S}_2(t)$$

According to Eq. B1 the change $\Phi(t + dt) - \Phi(t)$ in time dt is equal to $-iH\Phi(t)dt$, so each term in the Hamiltonian drives some part of the system. The Hamiltonian of this system in standard quantum mechanics is $H = H_1 + H_{12} + H_2$, where H_1 drives $S_1(t)$ by itself, H_2 drives $S_2(t)$ by itself, and H_{12} drives the interaction between the two. If we don't want to drive $S_2(t)$ dynamically on its own, we simply remove that term in the Hamiltonian. This gives

$$H = H_1 + H_{12}$$

as the Hamiltonian that results when applying nRule (4). Schrödinger's equation is then

$$H_1 S_1(t) + H_{12}[S_1(t) + \underline{S}_2(t)] = (id/dt)\{S_1(t) + \underline{S}_2(t)\} \quad (\text{B2})$$

The first term $H_1 S_1(t)$ in this equation provides for the internal evolution of $S_1(t)$ by itself. The second (square bracketed) term provides for the changes in square modulus of the two components; and in addition, it transmits internal changes in $S_1(t)$ to $\underline{S}_2(t)$. So $\underline{S}_2(t)$ is a function of time because its square modulus changes in time, and because the internal dynamical changes in $S_1(t)$ are continuously being transmitted to it. It continuously updates the boundary conditions of the new solution. However, it does not evolve on its own because H_2 is not present. These are the properties we want for the launch state.

Our major assumption is that there exists a unitary operator that displaces this system (with its truncated Hamiltonian) in time. That cannot be formally demonstrated. It is justified only by Eq. B2 that appears to give the desired result.

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