

The Cat nRules

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The nRules that are developed in another paper are applied to two versions of the Schrödinger cat experiment. In version I the initially conscious cat is made unconscious by a mechanism that is initiated by a radioactive decay. In version II the initially unconscious cat is awakened by a mechanism that is initiated by a radioactive decay. In both cases an observer is permitted to check the statues of the cat at any time during the experiment. In all cases the nRules correctly and unambiguously predict the conscious experience of the cat and the observer.

Introduction

Four rules called the *nRules* are given in a previous paper [1]. These rules are said to govern the process of stochastic choice and state reduction in an ontological model of a quantum mechanical system, and describe how conscious awareness of the observer changes (or not) during this process. In the present paper, these rules are applied to Schrödinger's cat.

The first of the *nRules* refers to the probability current J that flows into a state. Current J is the time rate of change of the square modulus.

nRule (1): *For any subsystem of n complete components in a system having a total square modulus equal to s , the probability per unit time of a stochastic choice of one of those components at time t is given by $(\sum_n J_n)/s$ where the net probability current J_n going into the n^{th} component at that time is positive.*

[**note:** A *complete component* is a solution of Schrödinger's equation that includes all the variables of all the (symmetrized) objects in the universe. See ref. 1 for further elaboration.]

The second rule specifies the conditions under which *ready states* appear in solutions of Schrödinger’s equation. These are understood to be the *basis states* of state reduction (i.e., collapse of the wave function in an individual case). Ready states are always underlined.

nRule (2): *If an interaction produces complete components that are discontinuous with the initial component, then all of the new states that appear in these components will be ready states.*

(see ref. 1 for elaboration on “discontinuous” and “initial”)

The third rule provides for the collapse of the wave.

nRule (3): *If a component containing ready states is stochastically chosen, then all those states will become realized, and all other components in the superposition will be immediately reduced to zero.*

[**note:** If a state in a complete component is not ready is called *realized*. Realized states are not underlined.]

(see ref. 1 for an elaboration on “immediately”)

nRule (4): *If a component in a superposition is entangled with a ready state, then that component can only receive probability current.*

The purpose of the present paper is to apply these rules to two versions of Schrödinger’s cat experiment. Version I assumes that the cat is initially conscious and is made unconscious by a mechanism that is initiated by a radioactive decay. In version II, the cat is initially unconscious and is made conscious by an alarm clock that is initiated by a radioactive decay.

The Apparatus

We first look at the apparatus that is used in Schrödinger’s cat experiment *without* a cat or an external observer being present. It consists of a radioactive source and a detector (denoted by either d_0 or d_1), where d_0 means that the detector has not yet captured the decay particle, and d_1 means that it has. The detector output will be connected to a

mechanical device that carries out a certain task, such as a hammer falling on a container that releases an anesthetic gas. This device is a function of time given by $M(t)$, where $M(t_0)$ is its configuration (e.g., the position of the hammer) prior to its being stochastically chosen. The component $d_0M(t_0)$ indicates that the radioactive source had not yet triggered the mechanism M . The component $d_1M(t)$ indicates that the detector has captured a decay particle and the mechanism has advanced to its position at time t . Let i_0 be an indicator that tells us that M has not yet completed its task, and i_1 tell us that it has. When $M = M(t_f)$ we will say the device has fully run its course, so $d_1M(t_f)i_1$ means the source has decayed and the mechanical device has completed its task as indicated by the time t_f and the indicator i_1 . We also suppose that the source is exposed to the detector for a time that is limited to the half-life of a single emission. At that time a clock will shut off the detector, so it will remain in the state d_0 if there has not yet been a particle capture.

The system is $\Phi(t_0) = d_0M(t_0)i_0$ at the beginning of the primary interaction at time t_0 when the detector is first exposed to the radioactive source. After that

$$\Phi(t \geq t_0) = d_0(t)M(t_0)i_0 + \underline{d}_1(t)M(t_0)i_0 \quad (1)$$

where the second component is zero at t_0 and increases in time. This component cannot evolve beyond t_0 because it is a ready state as required by nRule (2) and is therefore a dead-end because of nRule (4). We do not represent the source in this expression. Probability current flows from the first to the second component, so it is possible that there will be a stochastic hit on the ready state \underline{d}_1 at time t_{sc} giving

$$\Phi(t = t_{sc} > t_0) = d_1(t)M(t_0)i_0$$

where d_1 has been made a realized state by nRule (3). As the mechanical device subsequently completes its course at time t_f , we will have

$$\Phi(t_f > t > t_{sc} > t_0) = d_1(t_{sc})M(t_0)i_0 \rightarrow d_1(t_{sc})M(t)i_0 \rightarrow d_1(t_{sc})M(t_f)i_1$$

where the arrows denote a continuous evolution from t_{sc} to t_f . At and after t_f the state of the system is

$$\Phi(t \geq t_f > t_{sc} > t_0) = d_1(t_{sc})M(t_f)i_1 \quad (2)$$

If the half-life time $t_{1/2}$ of the source runs out before there is a stochastic hit, then eq. 1 will become time independent and the second component will become a *phantom*¹. The phantom can be dropped out of the equation and we are then left with

$$\Phi(t \geq t_{1/2} > t_0) = d_0(t_{1/2})M(t_0)i_0 \quad (3)$$

Fifty percent of the time the system will finish with $d_1(t_{sc})M(t_f)i_1$ in eq. 2, and the rest of the time it will finish with $d_0(t_{1/2})M(t_0)i_0$ in eq. 3.

Add Observer

If an observer looks at the apparatus during the time before there has been a stochastic hit (eq. 1), then prior to the observer's interaction we will have

$$\Phi(t \geq t_0) = \{d_0(t)M(t_0)i_0 + \underline{d}_1(t)M(t_0)i_0\} \otimes X \quad (4)$$

where X is the unknown state of the observer prior to interaction. Let the observer look at the detector at time t_{look} .

$$\begin{aligned} \Phi &= d_0(t_{\text{look}})M(t_0)i_0 \otimes X \rightarrow d_0(t_{\text{look}} + \pi)M(t_0)I_0B_0 \\ &+ \underline{d}_1(t_{\text{look}})M(t_0)i_0 \otimes X \rightarrow \underline{d}_1(t_{\text{look}} + \pi)M(t_0)I_0B_0 \end{aligned} \quad (5)$$

where B_0 is the observer's brain when it is conscious of the indicator I_0 . The capital I is an extended indicator that includes the original device i plus the low level physiological processes of the observer. The brain state B only includes the part of the brain that is explicitly involved with the conscious experience. The continuous evolution (arrows) in eq. 5 carries i into I and $\otimes X$ into B_0 in time π . In this treatment arrows always indicate a continuous evolution, whereas a plus sign indicates a discontinuous "quantum jump".

The first row in eq. 5 is a *single component* that evolves continuously in response to the physiological interaction. The primary interaction is still active during this time and that gives rise to a vertical current going from the first to the second row in eq. 5. The vertical evolution is a discontinuous jump in both cases. The second row is therefore a continuum of components that are created parallel to the first row at each moment of time. So at time $t_{\text{look}} + \pi$, vertical current flows only into the final component in the second row of eq. 5. Components prior to the last one no longer have current flowing

¹ A phantom no longer serves any purpose and can be dropped from the equation. This is like redefining the system at time $t_{1/2}$. For further discussion, see ref. 1

into them from above; and since there can be no horizontal current among these ready states, they become phantom components as soon as they are created². The time of the observation (i.e., when the physiological interaction is complete) is called $t_{ob} = t_{look} + \pi$, so eq. 5 is

$$\Phi(t \geq t_{ob} > t_0) = d_0(t)M(t_0)I_0B_0 + \underline{d}_1(t)M(t_0)I_0B_0 \quad (6)$$

where $\underline{d}_1(t)M(t_0)I_0B_0$ is zero at t_{ob} and increases in time. This is the same as eq. 4 with the observer on board.

With a stochastic hit on the second component in eq. 6 at time t_{sc} , we get

$$\Phi(t = t_{sc} > t_{ob} > t_0) = d_1(t)M(t_0)I_0B_0$$

As the mechanical device runs its course from t_{sc} to t_f we will have

$$\Phi(t_f \geq t \geq t_{sc} > t_{ob}) = d_1(t_{sc})M(t_0)I_0B_0 \rightarrow d_1(t_{sc})M(t)I_0B_0 \rightarrow d_1(t_{sc})M(t_f)I_1B_1$$

where the arrows denote a continuous evolution from t_{sc} to t_f . The brain state B_1 observes the indicator state I_1 . So prior to the stochastic hit, the observer in eq. 6 is conscious of I_0 ; and after the stochastic hit, the observer is first conscious of I_0 and then I_1 . There are two different brain states in this expression, but they occur at different times so there is no paradoxical ambiguity. At and after t_f the state of the system is

$$\Phi(t \geq t_f) = d_1(t_{sc})M(t_f)I_1B_1 \quad (7)$$

which says that the observer has come on board and sees the final state of the indicator I_1 .

If the clock runs out at $t_{1/2}$ before there has been a stochastic hit, then eq. 6 will become time independent and the second component will become a phantom. In that case

$$\Phi(t \geq t_{1/2} > t_{ob}) = d_0(t_{1/2})M(t_0)I_0B_0 \quad (8)$$

Fifty percent of the time the system will finish with $d_1(t_f)M(t_f)I_1B_1$ in eq. 7, and the rest of the time it will finish with $d_0(t_{1/2})M(t_0)I_0B_0$ in eq. 8.

If there should be a stochastic hit on eq. 5 in the middle of the physiological interaction (i.e., the continuous horizontal development in that equation), then the

² Any component in the second row is complete. It is continuous with itself in time, but not with any *other* component. It is also a ready component arising out of the primary interaction; and for this reason, it cannot undergo a continuous evolution on its own. So as soon as vertical current from the first row stops flowing into it, it becomes a phantom. However, new components arise that are ‘temporally’ continuous with it. The result is a continuum of ready states in the second row that are phantoms except for the one that still receives vertical current.

corresponding ready state in the second row will be chosen, and a subsequent classical/continuous evolution will carry the system all the way to eq. 7.

Version 1 with no Outside Observer

We now replace the indicator in eq. 1 with cat brain states, the first of which is the conscious state C shown in eq. 9 below. This says that the cat is initially conscious of the mechanical device in its state $M(t_0)$. The device is understood to render the cat unconscious when it reaches $M(t_f)$. In this case we require that all lower physiological operations of the cat's brain are included in mechanical device. Before a stochastic choice occurs, the system is given by

$$\Phi(t \geq t_0) = d_0(t)M(t_0)C + \underline{d}_1(t)M(t_0)C \quad (9)$$

where the second component is zero at t_0 and increases in time, and where the cat is entangled with the mechanical device from the beginning. The ready detector state \underline{d}_1 represents a capture of the radioactive decay, the source of which is not shown. If there is a stochastic hit on the second component at time t_{sc} , eq. 9 becomes

$$\Phi(t_f \geq t \geq t_{sc} > t_0) = d_1(t_{sc})M(t_{sc})C \rightarrow d_1(t_{sc})M(t)C \rightarrow d_1(t_{sc})M(t_f)U$$

where the arrows represent a continuous classical evolution from $M(t_{sc})$ to $M(t_f)$ and from the conscious state C to the unconscious state U . Again, there are two different brain states in this expression, but they occur at different times so there is no paradoxical ambiguity of the kind generally associated with Schrödinger's cat.

If there is no stochastic hit on eq. 9 by the time the interaction is cut off at $t_{1/2}$, then the second component will become a phantom, leaving

$$\Phi(t \geq t_{1/2} > t_0) = d_0(t_{1/2})M(t_0)C$$

So the cat will have escaped unconsciousness 50% of the time.

Version 1 with Outside Observer

We begin with eq. 9 except that there is now an outside observer X waiting in the wings

$$\Phi(t \geq t_0) = \{d_0(t)M(t_0)C + \underline{d}_1(t)M(t_0)C\} \otimes X$$

Let the observer look at the cat at time t_{look} .

$$\begin{aligned}\Phi &= d_0(t_{\text{look}})M(t_0)C \otimes X \rightarrow d_0(t_{\text{ob}})M(t_0)CB_C \\ &+ \underline{d}_1(t_{\text{look}})M(t_0)C \otimes X \rightarrow \underline{d}_1(t_{\text{ob}})M(t_0)CB_C\end{aligned}$$

where B_C is the observer's brain when it is conscious of the cat in its conscious state C . The mechanical device is now expanded to include the low level physiology of the outside observer. As in eq. 5, the second row is a continuum of ready components, only the last of which survives at t_{ob} to give

$$\Phi(t \geq t_{\text{ob}} > t_0) = d_0(t)M(t_0)CB_C + \underline{d}_1(t)M(t_0)CB_C \quad (10)$$

where the second component is zero at t_{ob} and increases in time. If there is a stochastic hit on the second component of this equation, we get

$$\Phi(t_f \geq t \geq t_{\text{sc}} > t_{\text{ob}} > t_0) = d_1(t_{\text{sc}})M(t_{\text{sc}})CB_C \rightarrow d_1(t_{\text{sc}})M(t)CB_C \rightarrow d_1(t_{\text{sc}})M(t_f)UB_U$$

where B_U is the observer's brain state when it is conscious of the unconscious cat in the ready state U . This equation says that at time t_f the cat is made unconscious and is observed in that state by the outside observer. In this case both the cat and the observer have different brain states in the same expression, but they both occur at different times so the result is not paradoxical.

If there is no stochastic hit on eq. 10 by the cut-off time $t_{1/2}$, then the second component will become a phantom, leaving

$$\Phi(t \geq t_{1/2} > t_{\text{ob}} > t_0) = d_1(t_{1/2})M(t_0)CB_C$$

indicating that the cat has escaped unconsciousness as observed by the outside observer.

Version II with no Outside Observer

In the second version of the Schrödinger cat experiment, the cat is initially unconscious and is awakened by an alarm that is set off by the capture of a radioactive decay. The mechanical device $M(t)$ is now an alarm clock. As before, it will go off when the device reaches $M(t_f)$, which happens only 50% of the time.

$$\Phi(t \geq t_0) = d_0(t)M(t_0)U + \underline{d}_1(t)M(t_0)U \quad (11)$$

If there is a stochastic hit on the second component, then

$$\Phi(t_f \geq t \geq t_{\text{sc}} > t_0) = d_1(t_{\text{sc}})M(t_{\text{sc}})U \rightarrow d_1(t_{\text{sc}})M(t)U \rightarrow d_1(t_{\text{sc}})M(t_f)C$$

so

$$\Phi(t \geq t_f > t_{\text{sc}}) = d_1(t_{\text{sc}})M(t_f)C$$

If there is no stochastic hit by the time $t_{1/2}$, then the second component in eq. 11 is a phantom and we get just

$$\Phi(t \geq t_{1/2} > t_0) = d_0(t_{1/2})M(t_0)U \quad (12)$$

Version II with Outside Observer

Starting again with eq. 11 with an outside observer standing by

$$\Phi(t \geq t_0) = \{d_0(t)M(t_0)U + \underline{d}_1(t)M(t_0)U\} \otimes X$$

The observer looks at time t_{look}

$$\begin{aligned} \Phi &= d_0(t_{\text{look}})M(t_0)U \otimes X \rightarrow d_0(t_{\text{ob}})M(t_0)UB_U \\ &+ \underline{d}_1(t_{\text{look}})M(t_0)U \otimes X \rightarrow \underline{d}_1(t_{\text{ob}})M(t_0)UB_U \end{aligned}$$

In the second row only the last component survives to give

$$\Phi(t \geq t_{\text{ob}} > t_0) = d_0(t)M(t_0)UB_U + \underline{d}_1(t)M(t_0)UB_U \quad (13)$$

which puts the observer on board with the unconscious cat. The second component is zero at t_{ob} and increases in time. If there is then a stochastic hit at time t_{sc}

$$\Phi(t = t_{\text{sc}} > t_{\text{ob}} > t_0) = d_1(t_{\text{sc}})M(t_0)UB_U$$

and subsequently

$$\Phi(t_f > t > t_{\text{sc}}) = d_1(t_{\text{sc}})M(t_0)UB_U \rightarrow d_1(t_{\text{sc}})M(t)UB_U \rightarrow d_1(t_{\text{sc}})M(t_f)CB_C$$

concluding in

$$\Phi(t \geq t_f) = d_1(t_{\text{sc}})M(t_f)CB_C$$

If there is no stochastic hit, then the second component in eq. 13 is a phantom and we are left with

$$\Phi(t \geq t_{1/2} > t_{\text{ob}} > t_0) = d_0(t_{1/2})M(t_0)UB_U$$

Version II with a Natural Wake-Up

Even if the alarm does not go off, the cat will wake up naturally by virtue of its own internal alarm clock. The internal alarm can be represented by a classical mechanism $N(t)$ that operates at the same time as the external alarm $M(t)$. The interaction runs parallel to eq. 11 and is given by

$$\Phi(t \geq t_0) = N(t_0)U \rightarrow N(t)U \rightarrow N(t_f)C \quad (14)$$

Taking the product of eqs. 11 and 14 at t_0 gives

$$\Phi(t = t_0) = d_0(t)M(t_0)N(t_0)U$$

after which

$$\Phi(t \geq t_0) = [d_0(t)M(t_0)U + \underline{d}_1(t)M(t_0)U][N(t_0)U \rightarrow N(t)U \rightarrow N(t_f)C]$$

where the cross product suggests a conflict between C and U states. To resolve this, we follow two possible scenarios. The first assumes that the external stochastic choice and decay occurs before the internal decay, and the second assumes that the internal decay occurs before the external stochastic choice and decay. The first of these gives

$$\Phi(t \geq t_0) = d_0(t)M(t_0)[N(t_0) \rightarrow N(t)]U + \underline{d}_1(t)M(t_0)[N(t_0) \rightarrow N(t)]U$$

Following a stochastic hit we have

$$\Phi(t > t_{ff} > t_f > t_{sc} > t_0) = [d_1(t_{sc})M(t_0)U \rightarrow d_1(t_{sc})M(t)U \rightarrow d_1(t_{sc})M(t_f)C][N(t_0) \rightarrow N(t)]$$

resulting in

$$\begin{aligned} \Phi(t > t_{ff} > t_f > t_{sc} > t_0) &= d_1(t_{sc})M(t_f)[N(t_0) \rightarrow N(t) \rightarrow N(t_{ff})]C \\ &= d_1(t_{sc})M(t_f)N(t_{ff})C \end{aligned} \quad (15)$$

where t_{ff} is the final time of the internal continuous development.

The second scenario is

$$\Phi(t \geq t_0) = [N(t_0)U \rightarrow N(t)U \rightarrow N(t_{ff})C][d_0(t)M(t_0) + \underline{d}_1(t)M(t_0)]$$

resulting in

$$\Phi(t \geq t_{ff} > t_0) = N(t_{ff})[d_0(t)M(t_0) + \underline{d}_1(t)M(t_0)]C$$

followed by a stochastic hit, giving

$$\Phi(t_f > t > t_{sc} > t_{ff} > t_0) = N(t_{ff})d_1(t_{sc})[M(t_{sc}) \rightarrow M(t) \rightarrow M(t_f)]C$$

resulting in

$$\Phi(t > t_f > t_{sc} > t_{ff} > t_0) = d_1(t_{sc})M(t_f)N(t_{ff})C \quad (16)$$

Both of these scenarios lead to the same conscious state in eqs. 15 and 16.

Reference

1. R. A. Mould, “Quantum Brain nRules”, arXiv: physics/ 0406014