

Quantum Boundaries in Relativity

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9 August 2007

Abstract

It is claimed in another paper that the collapse of a quantum mechanical state must be covariant in the least, and that this can only be achieved if the collapse takes place along a covariant surface. The obvious surface available for this purpose is that of the backward time cone of an event, suggesting a collapse of the Hellwig-Kraus variety. This collapse is widely believed to result in paradoxical causal loops when applied to non-locally correlated particles. However, paradox is removed and linear causality is assured when we use the coordinate free qRule foundation theory that is developed in another paper. The boundaries of collapsed states are shown to have very distinctive characteristics in Minkowski space.

Introduction

The collapse of a quantum mechanical wave to a stochastically selected eigenstate is an undeniable feature of the physics of individual systems. However, the collapse of a state along a $t = \text{constant}$ surface of an arbitrary coordinate system is unbelievable, inasmuch as nature does not recognize a surface that is so obviously constructed by humans. For this reason, a foundation theory must provide for the collapse of a wave along a covariant surface that is independent of coordinate representation.

State reduction (i.e., wave collapse) is here governed by a set of three *qRules* that are introduced in another paper [1]. The covariant surface chosen is that of the backward time cone of a collapse. This surface was originally proposed by Hellwig and Kraus [2], but the idea has been widely rejected because of the apparent existence of causal inconsistencies [3]. However, when the *qRules* are applied to the problem these inconsistencies disappear.

It is found in another paper that the initial state of a quantum mechanical system can be mapped into the conic surface of Hellwig and Kraus, and that this function is

propagated to successive conic surfaces by the same dynamic principle that normally applies to it [4]. We take the case of two non-locally correlated particles, for that is the system most used to illustrate the supposed causal loops. It is shown below that causality is very linear when this system is subjected to the qRules, and as a result there are *no causal loops*. We then look at the affect of a number of parallel as well as sequential state reductions in Minkowski space and identify some of their unusual characteristics.

Foundation qRules of Quantum Mechanics

Standard quantum mechanics adopts a number of rules that tell us how the Schrödinger equation works. First there is the Born rule connecting square modulus with probability. Then there are the von Neumann rules governing ensembles when there is a collapse of a wave upon measurement; and finally, there are Bohr rules of measurement including the need for a quantum mechanical system to encounter a macroscopic measuring device. These rules are part of the understanding of quantum mechanics that are rarely given their deserved status as “physical law” at a level with Schrödinger’s equation; and yet, they represent an important theoretical superstructure of rules and principles that form the foundation of quantum mechanics.

The three *qRules* proposed in Ref. 1 are a codification of these extra-dynamical requirements. They are different from and I believe superior to the usual (Copenhagen) way of dealing with this theoretical superstructure. Important features of the qRules are: (1) they apply to individual trials as well as to ensembles of trials, (2) they are valid independent of size (micro or macroscopic), (3) they allow *all* observers to be continuously included in the system without ambiguity, (4) they account for the collapse of the wave function without introducing new or using old physical constants, (5) they allow energy and momentum to be conserved in an individual collapse, (6) they target a system for collapse without regard to its mass, (7) in dense environments they provide a high frequency of stochastic localizations of quantum mechanical objects, and (8) they are formulated in a covariant language that makes them useful beyond non-relativistic quantum mechanics.

The qRules

It is the covariant feature of the qRules that is most important in this paper. A qRule state is independent of all representations including coordinates. The theory is not just covariant, but it is *trans-representational* – independent of *all* representations (e.g., coordinate, momentum, energy, spin, etc.). The only familiar variable of a qRule component is ‘time’, which can be thought of as proper time along some chosen world line. For example, a state consisting of a single electron e and a single proton p can be written as $U(t) = e \otimes p(t)$ indicating that the two are non-interacting, or as $U(t) = ep(t)$ which does not commit as to interaction between the two. It can also be written as a single atom a in the form $U(t) = a(t)$. In all these cases, every other particle or field in the universe is understood to be in product with the indicated system. We say that every component in these equations is *complete* in this sense, as well as the sense in which it is not just part of an expansion in some representation.

If an interaction between an electron and a proton produces a bound atom a , this process as a function of time t is given by the trans-representational qRule equation

$$U(t \geq t_0) = ep(t) + \underline{a}\gamma(t) \quad (1)$$

where γ is a photon that is possibly emitted when the atom is formed, carrying away excess energy. The second component is assumed to be zero at time t_0 and increases in time. Probability current flows from the first to the second component, thereby preserving square modulus. The underline appearing under one of the states in the second component means that it is a *ready* component that has no empirical significance, the occurrence of which is specified by the qRules. Non-underlined components are called *realized* components and have empirical significance. These rules also tell us that positive probability current flowing from a realized component into a ready component will cause a *stochastic hit* on the latter with a probability per unit time equal to that current divided by the total square modulus. If a stochastic hit occurs in Eq. 1 at time t_{sc} , then according to the qRules, a state reduction occurs yielding another qRule equation

$$U(t \geq t_{sc} > t_0) = a\gamma(t)$$

indicating that a stochastic quantum jump has occurred. This causes a collapse of the equation in Eq. 1 that changes the ready (non-empirical) component $\underline{a}\gamma(t)$ into the realized (empirically real) component $a\gamma(t)$, and reduces $ep(t)$ to zero. The total square modulus is not preserved in this reduction inasmuch as currents (not magnitudes) are normalized by the qRules.

The qRules therefore declare that the capture takes place at some definite time when one empirically real component is replaced by another, although that time is initially uncertain. The cross section may not be sufficient to insure that an atom will be formed at all; and in that case, Eq. 1 will remain in a final static sum of ep and $\underline{a}\gamma$ after the interaction is complete.

The above equations do not specify the atom's orbit but that is easily done. Let a_1 be the first orbit and a_2 be the second orbit, etc. Equation 1 becomes

$$U(t \geq t_0) = ep(t) + \underline{a}_1\gamma_1(t) + \underline{a}_2\gamma_2(t) + \dots \quad (2)$$

where probability current flows from the first component to each of the ready components to an extent that is governed by the cross section of each. These components are not an expansion in the energy representation, for they are each dynamically related to the first component via a current flow. A purely formal expansion such as $a = a_1 + a_2 + \dots + a_n$ will not appear in a qRule equation. Eigenstates will appear in a qRule equation only if they are *physically distinguishable* from one another by virtue of an entanglement with some other interacting system such as the above radiation field. Each component in Eq. 2 is 'complete' in the sense defined above.

A stochastic hit on one of the ready components in this equation will cause all the other components to go to zero according to the qRules. For a fuller definition of qRule components and examples of their application, see Ref. 1. That definition requires that each qRule component equals the square modulus of the corresponding wave component with all of its variables (except t) integrated out. Each is therefore a representation-free square modulus of a wave component.

Initial and Collapsed States

In Ref. 4 we say that the collapse of a state must be covariant in the least. It will not collapse along a spatial surface defined by arbitrary coordinates; but instead, it will collapse along the surface of the backward time cone that is here called a *conic* surface. This is the Hellwig-Kraus state reduction that has been widely dismissed as being causally problematic (see Ref. 3). This issue will be dealt in detail with in the next section. One consequence of this kind of collapse is that the newly collapsed state of the system must also be defined (initially) along the backward time cone of a measurement event. So initial states as well as collapsed states are defined in this way. We conclude that the most natural quantum mechanical state is not specified along a constant time plane in an arbitrary system of coordinates; but rather, it is along the conic surface of a backward time cone of some definite event in space-time. Therefore, each of the components in qRule equations such as Eq. 1 or Eq. 2 are understood to refer to the associated wave functions that are spread out along a conic surface; and that the natural dynamic principle is one that projects one of these conic states into another conic state whose vertex lies on a world line going through the original vertex event. Conic states evolve naturally into other conic states (see Ref. 4).

Non-Local Correlations

Claims regarding a Hellwig-Kraus causal ambiguity are usually advanced by referring to non-local correlations in one form or another. Consider a pair of particles p_1 and p_2 that are created at a decay event \mathbf{O} to become correlated in the spin zero state.

$$\Psi(p_1, p_2) = 2^{-1/2} \{p_1(\uparrow)p_2(\downarrow) - p_1(\downarrow)p_2(\uparrow)\} \quad (3)$$

The first particle moves to the left in Fig. 1 and the second particle moves to the right. Initially they both have an uncertain spin direction as indicated by the double vertical arrow along each path in that figure. Imagine that the first particle is measured to have spin-up at an event \mathbf{A} in Fig. 1, causing a state reduction. In that case p_2 will go spin-down when it intercepts the backward light cone of event \mathbf{A} at event \mathbf{b} . If p_2 is measured at some latter time at event \mathbf{B} , it will of course record spin-down. In the end, the result is the state $p_1(\uparrow)p_2(\downarrow)$. Capital bold face letters indicate events that ‘cause’ state reduction –

like measurements; and lower case bold face letters identify events that are not themselves the cause of reduction.

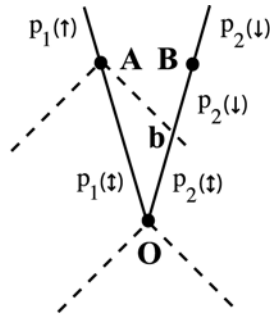


Figure 1: First non-local reduction

Event **B** also results in a state reduction and there are two possibilities. The effect of **B** on p_1 might be the same as the effect of **A** on p_2 as shown in Fig. 2a, where p_1 goes spin-up the moment it intercepts the backward light cone of event **B** at event **a**. So when p_1 is later measured at **A**, it would of course record spin-up. The final results would then be the same as those shown in Fig. 1. But this possibility revises the history of the first particle just prior to event **A** and leads to an odd circularity that is characteristic of a causal loop – i.e., **A** causes **b** causes **B**, and **B** causes **a** causes **A**.

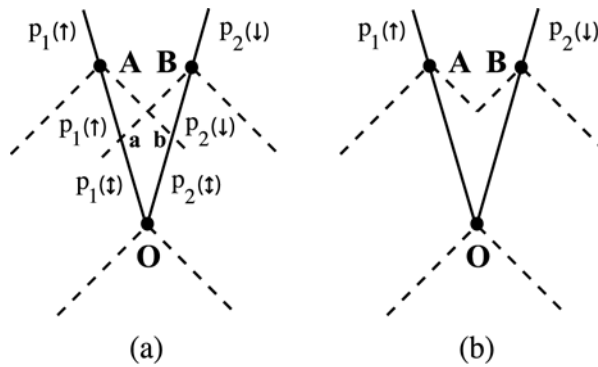


Figure 2: After second state reduction

This causal loop has led many to reject the Hellwig-Kraus state reduction scheme. Of course, historical revision by itself cannot violate causality if it has no net causal

effect, and it is clear that the effects of events **a** and **b** in Fig. 2a do not change the outcome. Nonetheless, the causal loop is disturbing.

There is another possibility. Perhaps only a double conic state like the double peaked dashed line in Fig. 2b will somehow survive the $p_1(\uparrow)p_2(\downarrow)$ measurement. We will see that this is a correct account of what happens, but the matter *cannot be decided on the basis of relativity or standard quantum mechanics alone*. We need the auxiliary rules that govern state reduction to make the case. In the following, the second possibility is derived directly from the qRules.

Apply the qRules

The particle pair is assumed to be created at an event **0** that we will say occurs at a time t_{00} along some preferred world line in Minkowski space. If the pair is created by the decay of a composite particle p_c , the corresponding qRule equation is given by

$$U(t \geq t_{00}) = p_c(t) + \underline{p}_1 p_2(t)$$

where $\underline{p}_1 p_2(t)$ is the qRule value of Ψ (in Eq. 3). It appears in this equation independent of representation. That's because it is the square modulus of Ψ with all of the variables except t integrated out. It is zero at $t_{00} = 0$ and increases in time. Probability current will flow from the first component $p_c(t)$ to $\underline{p}_1 p_2(t)$, leading to a stochastic hit and reduction at a time t_0 given by

$$U(t \geq t_0 > t_{00}) = p_1 p_2(t)$$

When the spin-measuring devices M_1 and M_2 of particle p_1 and p_2 are later introduced this becomes the qRule equation

$$\begin{aligned} U(t \geq t_0) = & p_1 p_2 \otimes M_1 M_2 m(t) + [\underline{p}_1(\uparrow) M_1] p_2(\downarrow) \otimes M_2 m(t) & (4) \\ & + \underline{p}_1(\uparrow) [p_2(\downarrow) M_2] \otimes M_1 m(t) \\ & + [\underline{p}_1(\downarrow) M_1] p_2(\uparrow) \otimes M_2 m(t) \\ & + \underline{p}_1(\downarrow) [p_2(\uparrow) M_2] \otimes M_1 m(t) \end{aligned}$$

where both measuring devices are on standby in the first component of Eq. 4, and the four 'ready' components (on the right) are zero at t_0 . In the ready component of the first row the first particle engages the spin-measuring device M_1 (square brackets), and in the second row the second particle engages M_2 . The third and fourth rows are similar except

that they provide for the reverse spin measurements. An independent molecule represented by m has been introduced into these equations so that we may follow its fate.

Probability current begins to flow from the first component to the ready components in the first and second rows when the first particle interacts with the measuring device M_1 . Current will begin to flow from the first component to the ready components in the third and fourth rows when the second particle interacts with M_2 . So all four ready components are exposed to the possibility of a stochastic hit. Figure 3a is a graphic description of Eq. 4. The gray area in that figure represents the Minkowski region of interaction where the ready components in Eq. 4 are activated.

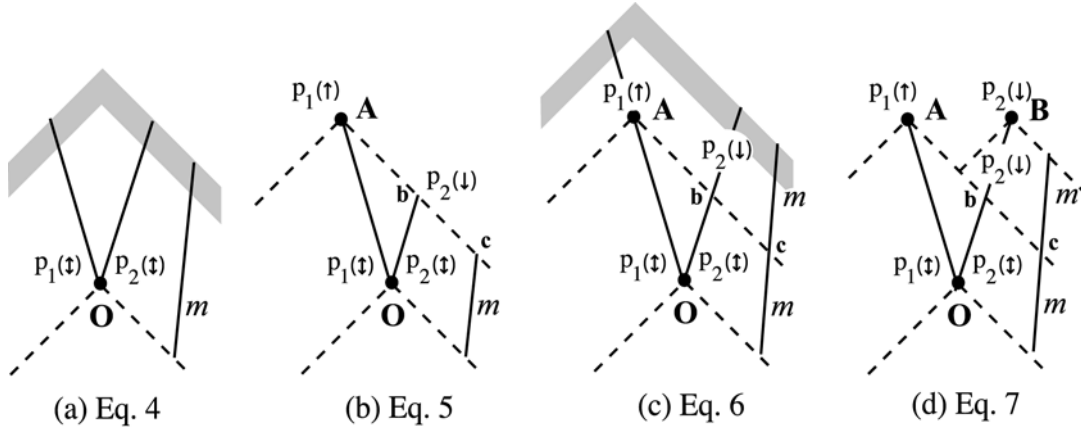


Figure 3: Non-local reduction

Suppose the first row is stochastically chosen at a time t_{scA} . This corresponds to event A in Fig. 1. The state is then reduced to

$$U(t = t_{scA} > t_0) = [p_1(\uparrow)M_1] p_2(\downarrow) \otimes M_2 m(t) \quad (5)$$

where t_{scA} is the time of the stochastic hit at event A that appears along the world line of p_1 (see Fig. 3b representing Eq. 5). It is understood that the wave functions representing p_1 , p_2 , M_1 , and M_2 in this figure are spread out along the conic surface of the vertex event A . On this surface event b is located at the interception point where p_2 first goes spin-down.

Further evolution carries the entire conic state forward along the world line following A until p_2 interacts with M_2 . A new ready state then emerges giving

$$U(t \geq t_{scA} > t_0) = [p_1(\uparrow)M_1] p_2(\downarrow) \otimes M_2 m(t) + [p_1(\uparrow)M_1] [p_2(\downarrow)M_2] \otimes m(t) \quad (6)$$

which is graphically represented in Fig. 3c in which the shaded area is the region of the new interaction that gives rise to the ready component in Eq. 6, when p_2 encounters M_2 .

Let a second state reduction occur at event **B** at time t_{scB} , giving the final result

$$U(t = t_{scB}) = [p_1(\uparrow)M_1] [p_2(\downarrow)M_2] m(t) \quad (7)$$

where t_{scB} is the time of the vertex event **B**. Equation 7 appears in Fig. 3d.

Since the qRules are covariant, they give us an *objective* account of what happens, and it is clear that there is a causal order to these equations. Equation 4 causally precedes Eq. 5 that causally precedes Eq. 6, and Eq. 6 causally precedes Eq. 7. We can therefore state the following corollary of the qRules.

Corollary: *The evolution of a qRule equation cannot affect the evolution of a previously ‘collapsed’ qRule equation.*

This means that the evolution described in Eq. 6 can have no influence on the evolution described in Eq. 4; for when Eq. 6 is in the process of evolving, Eq. 4 *no longer exists*. Of course the evolution of Eq. 4 leads to the realized state

$$[p_1(\uparrow)M_1] p_2(\downarrow) \otimes M_2 m(t)$$

in Eq. 5, so it can affect the evolution of Eq. 6. However, a causal influence cannot go in the other direction – from Eq. 6 to Eq. 4. It is the qRules that govern what happens, not just relativity or standard quantum mechanics, and these rules are causally clear.

The presence of the molecule m illustrates an important characteristic of conic state reductions. Its world line is cut off in Fig. 3b and its subsequent evolution in Fig. 3c reproduces some part of the evolution that previously occurred in Fig. 3a. This is because the time t in Eq. 4 refers to the vertex that appears at the top of the shaded region in Fig. 3a, whereas the time t_{scA} in Eq. 5 refers to the vertex at event **A** in Fig. 3b. So state reduction affects the molecule m as well as the primary participants p_1 and p_2 . The collapse doesn’t change m the way it does p_2 , but it does cause a *recreation* of m that results in a *repetition* of part of its evolution. Apparently history does repeat itself. But there is no still possibility of causal loops.

It may appear from the fate of m that everything in the universe will be recreated every time there is a collapse somewhere in the universe. That will not happen for

reasons that will be made clear. Basically it is because other reductions will block the influence of any one reduction.

After Eq. 7 the system will further evolve, possibly being exposed to another stochastic hit due to some interaction. This is shown graphically in the Minkowski space of Fig. 4, together with three other state reductions that may have happened in the vicinity. The more lightly shaded area (below **A** in Fig. 4) indicates the part of Minkowski space that evolves according to Eq. 4; and the more heavily shaded area (below **B**) indicates the evolution according to Eq. 6, where the darker area can have no influence on the lighter area because the latter is causally prior.

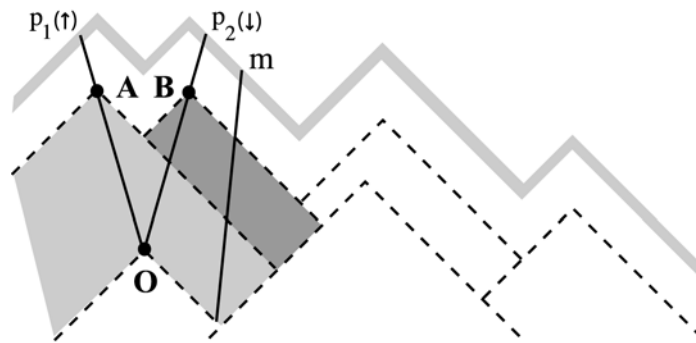


Figure 4: Causal order of multiple reductions

Apparently the prominent reduction that appears in the foreground of Fig. 4 is causally prior to all the others, for none of the other surfaces penetrate its backward time cone. Each one of these peaked areas represents the evolution and stochastic conclusion of a separate qRule equation. They fit together in an ordered causal sequence like the pieces of a four-dimensional jigsaw puzzle. Although the components included in each one of these puzzle pieces is complete (in that each one includes every particle in the universe) the reduced part of each component is limited in Minkowski space because it is contained by other (prior) qRule reductions. The lightly shaded envelope over the entire figure represents the introduction of a new ready component that arises from a new interaction that will lead to another collapse and a new mountaintop behind all the others.

Although the reduction at event **A** has recreated the independent molecule *m*, causing it to repeat some part of its evolution, the reduction does not recreate all of the

independent objects in the universe inasmuch as its influence cannot penetrate the backward time cones of the reductions that occur prior to it. In a dense material medium, this will put a serious limitation on the extent of any single collapse.

We do not say how the time t_{scB} in Eq. 7 is related to t_{scA} in Eq. 6. Figure 4 shows them to be the same time, and indeed for some world line (i.e., for some Lorentz observer) the two events do occur at the same time. If we choose another world line, event **A** will occur before event **B**, and for still another choice **A** will occur after **B**. So the temporal sequence of these events is a function of the observer in the usual relativistic sense. However, the qRules make a covariant statement that is independent of observers or world lines; namely, that Eq. 4 causally precedes Eq. 6. Apparently causal sequence is not the same as temporal sequence in this treatment.

That this conclusion is independent of one's choice of world line can be understood by realizing that the square modular current flow in a time dt is an invariant: $Jdt = \text{inv.}$ This means that the probability of a stochastic hit on any one of the four components in Eq. 4 during a time dt is *independent* of the chosen parameter t . For a Lorentz observer $J \propto (1 - \beta^2)^{1/2}$, where β describes the world line between event **0** and the potential stochastic event in question. The probability of a stochastic hit (which is proportional to Jdt/σ , where σ is the total square modulus) is therefore independent of the Lorentz frame. This is a minimum requirement for a relativistic quantum mechanics.

Although the term 'invariance' is used above, these results are understood to be entirely covariant. Invariance generally refers to the Lorentz group whereas covariance has a more general meaning. It may be convenient to couch the above argument in familiar Lorentz terms, but the intent is to make statements through the qRules that transcend coordinate representations entirely – and indeed all representations.

References

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