

Localization & the nRules

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8 March, 2006

The auxiliary nRules of quantum mechanics developed in previous papers are applied to the problem of the location of material objects – both macroscopic and microscopic. All objects tend to expand in space due to the uncertainty in their momentum. The nRules are found to oppose this tendency in two important cases that insure the dependable localization of objects in ordinary human experience.

Introduction

The auxiliary rules of quantum mechanics are rules that accompany Schrödinger's equation, relating it to the observation of individual processes in physics. The rules that have long been a part of standard quantum mechanics are referred to here as the *sRules*. They include the Born rule and others, such as the rules that govern what happens during measurement.

Alternative auxiliary rules that the author proposed in previous papers do not include the Born rule. Instead, probability is introduced into quantum mechanics through the *probability current* only. There are two rule-sets of this kind called the nRules [1, 2] and the oRules [3, 4]. In this paper we only consider the nRules. The four nRules listed in Ref. 2 are shown to successfully describe a wide variety of empirical situations. They are:

nRule (1): *If an irreversible interaction produces a complete component that is discontinuous with its predecessor in some variable, then it is a ready component.*

[**note:** This rule defines a *ready* component.]

[**note:** A *complete component* includes every (anti)symmetrized particle in the universe. A single particle is also complete in the sense that it is not a partial expansion in some representation.]

nRule (2): *A systemic stochastic trigger strikes a ready component with a probability per unit time equal to the positive probability current J/s flowing into it. A non-ready component is not stochastically chosen.*

[**note:** J is modular current and s is the total square modulus of the universe]

nRule (3): *When a ready component is stochastically chosen it becomes a realized component, and all other components go to zero.*

[**note:** a *realized component* is one that is not a ready component.]

nRule (4): *A ready component cannot transmit probability current to other components or advance its own evolution.*

[**note:** This is enforced by requiring that the part of the Hamiltonian whose variables refer only to a specific ready component does not drive that component. It only drives the realized component after a stochastic reduction.]

The Problem

A quantum mechanical object has an unavoidable uncertainty in momentum arising from the Heisenberg uncertainty principle. We therefore expect any object of any size to expand in time, taking up a larger and larger volume of uncertain location. Even a very large object, given enough time, would develop a significant uncertainty of position in its center of mass. There is nothing about Schrödinger's equation that would limit or reverse this process; and there is nothing about the sRules that would put an effective brake on this expansion. Even the earth would occupy a fuzzy orbit about the Sun together with its many macroscopic passengers; and this belies our common experience of our being well localized¹.

¹ The rate of expansion of any fully formed macroscopic object is infinitesimal. But every such object began as a widespread collection of hydrogen atoms following recombination about 14 billion years ago. Schrödinger's equation can provide the correlations that guide the subsequent evolution of that object, but it cannot 'locate' the object with greater accuracy than its original collections of hydrogen atoms. To do so requires the help of auxiliary rules that perform that 'state reduction' or localizing task. Copenhagen addresses this problem with an auxiliary rule that is circular because it uses macroscopic instruments to locate quantum mechanical objects without first locating those instruments.

Traditional measurement theory claims that position can be narrowed as a result of a position measurement, assuming that the measuring device is itself well localized. However, in a world in which everything has a quantum mechanical spread to it, a position measurement need not narrow the spatial distribution of the thing being measured *or* the spatial distribution of the measuring device. Standard quantum mechanical measurement only establishes a correlation between the two. It does not by itself bring about a definite location of either one.

It is the purpose of this paper to show that the auxiliary nRules resolve the problem that arises when there is no highly localized measuring device. These rules bring about reductions that restrain and generally oppose the Heisenberg expansion.

Case I – Standard Theory Localization

Assume that we have an object whose center of mass is quantum mechanically widespread, where the object given by $\psi(r, t)$ occupies a volume $V(t)$ at a time t . Normally $V(t)$ would expand in all directions because of its uncertainty of momentum along any given direction. We will take a picture of this object at some time t_0 using the highly localized camera in Fig. 1. The size of the object does not matter to this example.

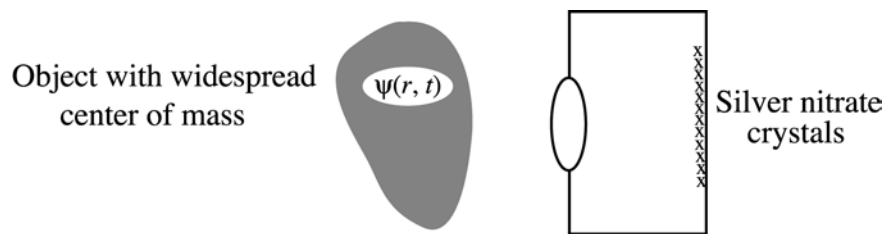


Figure 1: Photographing a quantum mechanical object

Imagine that just one photon reflects off of the object, enters the camera and interacts irreversibly with one of the silver nitrate crystals that make up the film. The initial state of the system is given by $\psi(r, t)C(t)$, where $C(t)$ is the camera with its unexposed film. The shutter is opened at t_0 after which the state equation is

$$\Phi(r, t \geq t_0) = \psi(r, t)C(t) + \sum_N \psi_N(r, t)\underline{C}_N(t) \quad (1)$$

where the summation components are zero at t_0 and increases in time. The state $\underline{C}_N(t)$ is a camera with its N^{th} crystal darkened, and $\psi_N(r, t)$ is the reduced state function of the object that is correlated with this value of N . The exact nature of $\psi_N(r, t)$ is not determined by the nRules, but by Schrödinger's equation.

If one did not apply auxiliary rules at all, the process would end here. Schrödinger's equation by itself cannot make stochastic choices among the components in the summation. Standard quantum mechanical sRules generally provide for a reduction following Eq. 1 because the macroscopic camera $C_N(t)$ will force a collapse of the wave into one component or the other. The nRules also force a collapse, but for different reasons.

Each component in the summation is 'ready' according to the nRules, so it contains the boundary conditions of the new solution of the Schrödinger equation that is launched when one of these components is stochastically chosen. Each of these launch components is frozen (according to nRule 4) until there has been a stochastic choice (see Refs. 1, 2). Each is designated by underlining one of its states, such as \underline{C}_n in Eq. 1.

Probability current flows from the first component in Eq. 1 into each of the summation components. If the a^{th} component is stochastically chosen at time t_{sc} according to nRule (2), a new collapsed solution will be launched (following nRule 3) giving

$$\Phi(r, t \geq t_{\text{sc}} > t_0) = \psi_a(r, t) C_a(t) \quad (2)$$

where the function $\psi_a(r, t)$ is the reduced state of the original function $\psi(r, t)$. So the effect of this irreversible and discontinuous interaction is a reduction in the spatial distribution of the object – opposing its natural tendency to expand under the influence of the uncertainty relationship.

Case II – Uncertain Film

In Case I we assumed that the camera was well localized, but that may not be the case. Although it is macroscopic, the camera might also have an uncertain location that increases in time because of its uncertainty of momentum. To simplify the discussion, we first assume that the camera has only two superimposed components. In Fig. 2 these components A and B are shown, where one is slightly off-center from the other.

Correlations are preserved in each component, so the silver nitrate crystals have the same relationship to the lens (and to each other) in component *A* as they do in component *B*. Prior to t_0 , there are no correlations between the camera and the object being photographed.

When the shutter is opened at t_0 the solution becomes

$$\Phi(r, t \geq t_0) = \psi(r, t)C_A(t) + \psi(r, t)C_B(t) + \sum_N \psi_{AN}(r, t)\underline{C}_{AN}(t) + \sum_N \psi_{BN}(r, t)\underline{C}_{BN}(t) \quad (3)$$

where the summations are zero at t_0 and increase in time. A photon leaving the object may not be absorbed by one of the crystals; but if it is, the stochastic trigger will hit on only one of the ready components in Eq. 3 (nRule 2). Suppose it selects $C_{Aa}(t)$ representing the $(Aa)^{\text{th}}$ crystal, then there will be a collapse of the state (nRule 3) yielding

$$\Phi(r, t \geq t_{sc} > t_0) = \psi_{Aa}(r, t)C_{Aa}(t)$$

Not only has the object been reduced from $\psi(r, t)$ to $\psi_{Aa}(r, t)$, but the crystal/camera state has also been reduced from the total number of crystals spread over two components of the superposition, to just one.

We are assuming that these crystals exist in a rich and random environment, for only then will the components *A* and *B* in Fig. 2 be truly independent of one another through decoherence. It is only because of their phase independence that complete eigenstates of the crystal/camera system can be localized separately in *A* and in *B*.

There is something artificial about this case. In the first place, the separation into only two components *A* and *B* is not very realistic; and in the second place, we cannot assume that the locations of either *A* or *B* are themselves well defined. We really need to know what happens when the line of crystals stretches continuously from *A* to *B*.

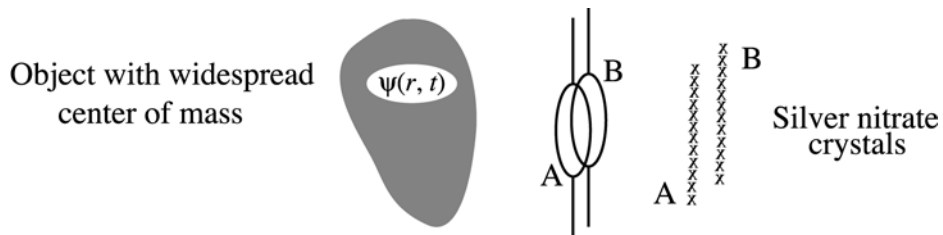


Figure 2: Quantum camera and a quantum mechanical object

Case III – Continuously Uncertain Film

To simplify this example, imagine that the camera film consisting of a *single crystal* of uncertain position lies along the single line shown in Fig. 3. The crystal and the camera are again correlated, so the position of the camera is also uncertain along the same line. There is now a continuum of camera/crystal components in superposition from which to choose.

Without the environmental influence, the initial system given by $\psi(r, t)C(t)$ would evolve into a single superposition of inseparable object and camera variables along the continuum in Fig. 3. However, in the presence of a rich and random environment, the phase relationship between different parts of the crystal/camera line would be disrupted, causing decoherence between different parts of the line. Suppose the crystal/camera components represented by the bottom three lines in Fig. 3 continue to form a coherent *batch* of superimposed components that is decoherent with the rest of the line. Also, let components represented by the top two lines form another coherent batch that is decoherent with the rest of the line. Then there will be three independent batches of crystal/camera components, where the system can be found fully contained in any one of these batches. In that case the state equation prior to a stochastic hit would be

$$\Phi(r, t \geq t_0) = \psi(r, t)C(t) + \sum_n \psi_{C_n}(r, t) \tag{4}$$

where the function $\psi_{C_n}(r, t)$ represents n^{th} batch which is an inseparable superposition of object and camera variables. The summation is zero at t_0 and increases in time. In the case considered, $n = 3$.

With a stochastic hit on the a^{th} batch at time t_{sc} , we get

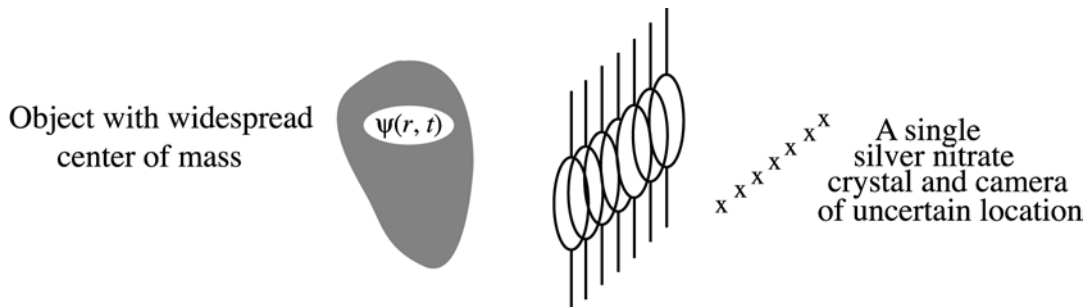


Figure 3: Quantum crystal/camera and a quantum mechanical object

$$\Phi(r, t \geq t_{sc} > t_0) = \psi \underline{C}_a(r, t)$$

So the collapse reduces the *object state* from the initial $\psi(r, t)$ to the a^{th} location, and at the same time it reduces the correlated *camera state* to that same location. Equation 4 fills in the gap between the two distinct components A and B in Eq. 3, and is certainly a plausible extension of that configuration. The single crystal example in Fig. 3 can easily be generalized to the Fig. 2 line of many different crystals².

The question may arise as to what happens if two batches are only partially decoherent from one another. Partial batches may indeed be independent. The only requirement is that eigenstates of the atom are found in each batch separately, giving $\psi \underline{C}_i(r, t) + \psi \underline{C}_k(r, t)$ for batches i and k . Until that degree of decoherence is reached, these components cannot be separated out; and beyond that point, the reduction is the same as it is when decoherence is complete.

Finally, consider Fig. 3 when the single crystal is a continuum of decoherent states that are bombarded by many photons. The moment any one of these states undergoes an irreversible darkening it will become a ready state, and the first one of these to be stochastically chosen will induce a localizing wave collapse. For a large number of photons that choice would be virtually instantaneous, even with a single crystal on the film. So for classically many photons on a typical film, the time for a location or wave collapse is essentially the time that it would take for one of the crystals on the film to undergo an irreversible chemical change.

Scattering

Let a single photon raise an atom to an excited state, only to drop down again by releasing a scattered photon. We focus on the spontaneous emission that follows the absorption of the incoming photon, and ignore possible stimulated emission and absorption. The incoming photon pulse is assumed to be spread-out widely over space.

² The theory of Ghirardi, Rimini, and Weber would also reduce the location of the microscopic object in Eq. 4, but by another process of “spontaneous reduction”, but that would only happen very rarely [5].

Let the atom also be spread over space by an amount that far exceeds its *minimum volume*. This is defined to be the smallest volume that the atom can occupy consistent with its initially given uncertainty of momentum. The atom (shaded area in Fig. 4) has had time to spread out prior to its interaction with the photon.

What will Schrödinger's equation do in this situation? As the incoming photon passed over the enlarged atom, we assume that the scattered photon will appear as a superposition of many scattered photons that are centered on different parts of the atom's extended volume as shown in Fig. 4 – these are the small wavelets in the figure. The correlations between the nucleus of the atom and the orbiting electrons must be preserved, even though the center of mass of the atom is spread out over a much larger volume. That is, the smaller dimensions of the minimal volume atom must be unchanged during its expansion, so the potential energy of the orbiting electrons is unchanged. The atom could not otherwise act as the center of a 'characteristic' photonic emission. This means that the incident photon having a characteristic frequency of the minimum volume atom will engage the enlarged atom throughout its volume.

However, if the atom is in a rich and random environment, its volume in Fig. 4 will not remain completely coherent. Imagine that it breaks up into n batches of lesser volume that are decoherent relative to one another, like the crystal/camera batches of Fig. 3. We will then have

$$\Phi(r, t \geq t_0) = \gamma(t)A(r, t) + \sum_n \gamma_n(r, t)\underline{A}_n(r, t)$$

where the summation is zero at t_0 and increases in time. The state $\gamma(t)$ is the not yet interacting incoming photon, $A(r, t)$ is the extended atom (total shaded area), $\gamma_n(r, t)$ is the photon emitted from the n^{th} batch, and $\underline{A}_n(r, t)$ is the ready atom associated with the n^{th} batch. In these circumstances, the emitted radiation from one batch will no longer be

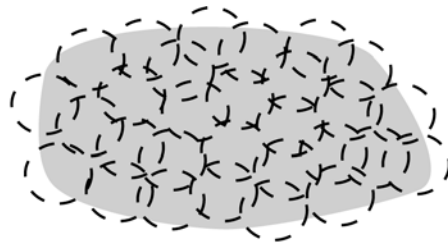


Figure 4: Scattered wavelets in superposition

coherent with the emitted radiation from another batch, indicating that each (potentially) contains an independently radiating atom. Very likely, the extent of decoherence that is necessary to isolate one batch from another is that which succeeds in making their radiations incoherent.

After a stochastic hit at t_{sc} ,

$$\Phi(r, t \geq t_{sc} > t_0) = \gamma_a(r, t)A_a(r, t)$$

Therefore, *the atom is reduced* to the size of the a^{th} batch. This reduction is not possible in any of the standard sRules (or unlikely in the GRW/CSL case) because it does not involve a macroscopic object.

Conclusion

We have now applied the nRules to the case of a camera taking a picture of an object in Fig. 3, and to the case of scattering in Fig. 4. There are undoubtedly many more examples involving irreversible chemical reactions resulting from discontinuous quantum jumps in a rich and random environment. But these two examples should be sufficient to guarantee that the ordinary macroscopic world that surrounds us is highly localized, despite its quantum mechanical tendency to expand. The sun's light entering the atmosphere will produce widespread scattering events that will continuously localize the atmosphere, and hence the correlated surface of the earth. The same will be true of the scattering of the Sun's light as it enters the waters of the earth's lakes and oceans. In addition, the biosphere includes many species that have eyes constructed along the lines of a camera. Here too, processes like one in Fig. 3 must be widespread, insuring a localization of the observer and the observed.

The nRules were originally formulated by considering macroscopic interactions by themselves. To this extent they function like an empirical formula that governs quantum mechanical observables. Applying these rules to any 'microscopic' process is therefore a matter of speculation, but one that appears to be plausible as has been demonstrated in other papers (Refs. 1, 2). In still another paper it is shown that the nRules settle the ambiguity long associated with the direction of time's arrow [6]. In the present paper we take speculation a step further. We assume that decoherence separates

otherwise coherent components for stochastic reduction. The resulting nRules not only describe microscopic and macroscopic phenomena, they also provide for the reliable localization of all observable things on our planet.

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