Covariant State Reduction

Richard A. Mould

Department of Physics and Astronomy, State University of New York,

Stony Brook, N.Y. 11794-3800

10 August 2007

The collapse of the quantum mechanical wave function associated with measurement is not automatic. A "wave" is usually given in a particular representation that lacks the objectivity of covariant description. However, there are rules and collapse equations in which all representations have been integrated out, so they provide a naturally covariant description of a collapse. But these leave no reference back to any representation. Therefore, recording the collapse of a wave in some particular representation – coordinate or otherwise – must be done by hand.

Physics has only succeeded in defining states over a limited range of representations, so it is possible that the universe makes no use of representations at all. In that case physical states would be defined in a way that is intrinsically covariant, and their collapse would be given in a language that avoids coordinate-bound differential equations.

Introduction

The wave function of a quantum mechanical system consisting of an independent atom, particle, and macroscopic object can be given by

$$\Phi(x_a\ldots,\ x_p \ldots x_m\ldots, t \geq t_0) = \psi_a(x_a \ldots, t)\psi_p(x_p, \ldots, t)\psi_m(x_m, \ldots t)$$

where x_a, x_p, x_m are the variables of the separate parts. The variables might themselves be basis vectors so Φ is considered a representation.

My contention is that universe makes no use of representations apart from an interaction in which their variables play a direct role. I believe representations are otherwise a purely human way of characterizing physical states, although they have been used with great success. We have found a dynamic process that works well for a limited number of coordinate representations, but certainly not for *all* such representations. The Hamiltonian of quantum mechanics does not work for generalized coordinates. The best we can do is confine the theory to the Lorentz group that gives a very narrow coordinate range. The field equation of general relativity is also not completely covariant because it only accepts coordinates that are continuous out to the second derivative. The equation does not allow discontinuous coordinates, so the theory is not invariant with respect to *all* transformations as is generally claimed. The best we can say is that the field equation of general relativity is covariant with respect to all coordinate transformations that 'work', but not to those that don't work.

We have had considerable success using representations, limited though they are, and perhaps that is the best we can do. But there is no reason to believe that nature is so constrained. There is no reason to believe that nature formulates its dynamic principle with respect to a limited set of representations, or that it relies on a coordinate group that is required to be continuous out to the second derivative. Nature is more likely to rely on specifications of another kind to impose lawfulness, although the detail of how this is done may be presently beyond us. Nonetheless there are things we can now do at this level.

Trans-Representation States

Let $U(t)$ be a function of time that refers (in this case) to an atom a, a particle p, and a macroscopic object m, and is written

$$U(t) = apm(t) \tag{1}$$

Equation 1 makes no statement about the location of these parts, or their possible entanglement or ongoing interaction. This information is assumed to be independently given without using wave variables. For instance, the universe might locate an object in space-time by simply *putting it there* without attaching coordinate numbers to it. Only metrical relationships to other objects would be needed. A spin state might be defined by giving it spin values with or without its orientation with respect to other objects. And the interaction between the parts of any two of the objects in Eq. 1 might be specified by their proper metrical separation together with appropriate constants. This sparse notation need not lose the interference properties that we associate with waves. For a given distribution of a particle over a region of space-time it is possible to say that there is a coherent phase difference between its different parts, and that that can be stated in a covariant language that makes no reference to coordinate values. The manner of these specifications is not precisely defined in this paper and may not be definable with any precision using recognizable notation. The only recognizable variable in Eq. 1 is the proper time along a particular world line.

The spatial distribution of an object at time t_0 will be advanced in time by the action that presumably gives $U(t)$ in Eq. 1. This equation will be constant for a continuous temporal change. However, $U(t)$ will clearly reflect the change of a system undergoing a quantum jump or *state reduction* (i.e., collapse) after a time t_0. Leading up to a collapse we will have

$$U(t \geq t_0) = apm(t) + \underline{a}m'(t) \tag{2}$$

where the particle is shown to be irreversibly captured by the macroscopic object *m*, so it becomes a part of that instrument described by *m*'. The second component in Eq. 2 is initially zero and increases in time as probability current flows to it from the first component, thereby preserving the given normalization. The second component is underlined to distinguish it from the first component – the latter being objectively real at the indicated time. The second component is *not* objectively real *until* it is stochastically chosen at time t_{sc}, at which time the first component (or what is left of it) goes to zero and the equation becomes

$$U(t \geq t_{sc} > t_0) = am'(t) \tag{3}$$

The surviving component is real in this equation so it is no longer underlined. Equations 2 and 3 represent the collapse of the wave that is generally associated with a measurement. The collapse of a quantum mechanical state is an covariant thing that takes place *across all representations* of the complete state. Therefore, it can be described by equations like Eqs. 1-3, the “verbal” aspects of which are sufficient for present purposes.

Consider another example. Let an electron interact with a proton where there is a determined cross-section of capture in different energy states a_1, a_2, and a_3, etc. of the atom *a*. This is written

$$U(t \geq t_0) = ep(t) + \underline{a}_1\gamma_1(t) + \underline{a}_2\gamma_2(t) + \ldots \quad (4)$$

where e is the electron, p is the proton, and γ is the (possible) photon emitted in each case. Each of the underlined (unreal) components is initially zero and receives probability current from $ep(t)$, causing them to increase in time. Each is actually a continuum of possibilities representing the different ways in which excess energy might be thrown off during the capture (e.g., inelastic collision with or without gamma). A stochastic hit on any one of the underlined components will lead to a collapse of the state to the component of choice. Equation 4 is *not* an expansion in the atomic energy representation. It is a *trans-representational equation* in which each energy component exists by virtue of a finite cross-section, and each receives probability current based on that cross-section. So energy eigenstates are not important because of a possible expansion of the state in the energy representation, but because of their critical role in energy-selective interactions of this kind.

The qRules

The general rules that govern any quantum mechanical collapse are given in another paper [1]. They are called qRules, and equations like Eqs. 1-3 are called qRule equations. These rules tell us how to derive qRule equations from a given wave function; but if given qRule equations, it is not possible to go in the other direction and derive the corresponding wave equation. That's because qRule equations are found by integrating out all of the variables of the representation in which the wave function is given. That process is not reversible unless one knows how to restore the variables in the representation of choice. However, our contention is that nature does not make choices of that kind, so nature cannot describe a collapse in terms of conventional Schrödinger solutions. Therefore, if you or I want to follow a collapse of the wave in some special representation, we are on our own. It turns out that the qRules often give good results when applied directly to Cartesian solutions to the wave equation. But for a thoroughly consistent foundation theory, the rules should be formulated in terms of more abstractly specified trans-solutions like $U(t)$.

The qRule theory has *experimental consequences* that distinguish it from other foundation theories. It predicts that qRule state reduction (collapse) results directly or indirectly from collisions as is shown in Ref. 1. Reduction under these rules therefore has the unique property of being a function of temperature and pressure. An example is given in Ref. 1.

Physics has been enormously successful using the familiar representations in both relativity and quantum mechanics. Surely this will continue to be the case. However, our reliance on classical mathematics, and coordinate-bound differential equations in particular, to push further into the mysteries of the universe might have its limitations. We may still be able to make foundational statements that are totally independent of representations – like the qRules that govern the invariant collapse of the wave function; but the kind of detail description of states and their dynamic change that we are accustomed to might not be possible at the fundamental level.

References

1. R. A Mould, "Quantum qRules: Foundation theory", presently on the author's web site http://ms.cc.sunysb.edu/~rmould/index.html