POL 606 – Time Series Analysis
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Week 11
Dynamic Conditional Correlations

DCC is another method for studying time varying parameters.

Allows for the study of context – under what conditions does a theory apply?

DCC calculates a current correlation between variables of interest as a function of past realizations of both the volatility within the variables and the correlations between them.

Thus, the relationship between variables can be seen to evolve over time in a manner that not only depends upon whether and to what degree the variables are moving in the same direction, but also takes account of the history of variance that each series has undergone.

Based within the family of Generalized Autoregressive Conditional Heteroskedasticity (GARCH; see Engle 1982; Bollerslev 1990) models.

An inability to study the importance of context sets traditional time series analysis behind cross-sectional analyses, which can use tools such as interaction terms to study how additional factors affect the connection between independent and dependent variables.

An initial question we may wish to answer is: "What is the correlation between two series *now*?"

One way to answer this is estimate a correlation using all the data we have available to us.

However, this will include old information that may be of far less use to us than recent information.

On the other hand, using only a few recent observations – say the last 10 – will create more variability because of the small sample size. Further, we will be assigning zero weight to older observations that may be worth including.

Moving-window analyses, also referred to as "rolling correlation estimators," are a middle ground here.

One specifies some length of time s where $s < T$, the full sample size. The correlation or regression coefficient is then estimated over the period 1 to s, then 2 to $s+1$, and so on until $T-s+1$ to T. These correlations provide some information on the evolution of the relationship between variables. Martin (1998) points out some disadvantages to this approach: the user must adopt an *ad hoc* approach to choose window width and moving-window analyses cannot account for abrupt changes in volatility very well. Beck (1983) argues that this approach can give unstable estimates and offers no statistical test. Another drawback is that it equally weights all observations less than s periods in the past and gives no weight at all to older observations (Engle 2002). Further, suppose we have yearly data from 1901-2000 and use a moving window regression that is thirty years wide. Doing so, the years 1901 and 2000 appear in one regression equation each, the years

DCC approach allows series to have periods of positive, negative, or no correlation.

For example, perhaps presidential approval is tightly tied to the economy, public opinion about foreign policy, concern about crime, etc. when that issue is prominently placed on the issue agenda (whether by elites, the media, etc.) and not at other times.

Both direction of the correlation and strength of the correlation can be considered.

When two series move in the same direction, the correlation increases and is positive. When they move in opposite directions, the correlation is decreased and may become negative.

The effects of the economy or foreign policy are likely to be stronger during periods of elections and war, respectively. Whether or not there is an equilibrium that the series return to is also interesting to consider.

Modeling Procedures

Autoregressive conditional heteroskedasticity (ARCH) models and the generalized extension (GARCH) have been very successful in modeling time-varying variances (Tse 2000; Gronke and Brehm 2002).

Extensions have moved from the univariate to multivariate setting. An important assumption for determining which multivariate modeling extensions are appropriate involves testing whether the assumption of constant correlations holds, i.e., simply whether the correlations are time invariant or not.

If they are time-varying, we then proceed to examine the interaction of multiple series by using Engle's (2000) dynamic conditional correlation multivariate GARCH Model, which is referred to as the DCC model for short.

The method estimates the DCC parameters and the time-varying conditional correlations among the returns.

The estimates of correlation can then be used to analyze significant events that occurred as well as the impact of the other series (Bautista 2003).

Testing for Constant Correlations

Tse (2000) provides a convenient and straightforward Lagrange Multiplier (LM) test for the assumption of constant correlations in a multivariate Generalized-ARCH (GARCH) model.

Thus, the Tse test is a useful first step to establish that varying correlations are statistically significant and that a DCC estimator is warranted.

Operationally, Tse (2000) specifies a constant correlation model as a dynamic one with certain key restrictions imposed on the model. Specifically, for time series Y_1 to Y_k with conditional variances is given by:

$$\sigma_{it}^2 = \varpi_i + \alpha_i \sigma_{i,t-1}^2 + \beta_i y_{j,t-1}, \; i = 1,...,K$$

We have time varying covariances of:

$$\sigma_{ijt} = \rho_{ij} \sigma_{it} \sigma_{jt}, \; 1 \le i < j \le K$$

and time varying correlations of:

$$\rho_{ijt} = \rho_{ij} + \delta_{ij} y_{i,t-1} y_{j,t-1}.$$

A test of the null hypothesis that δ_{ij} =0, establishes whether the correlation between two series is dynamic – that is, do they have constant correlations or are they time varying?

While the equations given pose the test for a GARCH(1,1) test, the Tse test is easily adaptable to GARCH(p,q) models and the same logic is applied.

The DCC Model

If the correlations are dynamic and not constant, the next step is to model the series in a multivariate setup.

The correlations can be measured and predicted, as well as the volatility.

Estimation of DCC is broken into two stages, which simplifies the estimation of a time varying correlation matrix.

This can be done in one step but it becomes very difficult with more than 2 variables in RATS and near impossible in any program with 4 or more variables.

In the first stage, univariate volatility parameters are estimated using GARCH models for each of the variables.

In the second stage, the standardized residuals from the first stage are used as inputs to estimate a time-varying correlation matrix. Two-step estimation of the likelihood function means that estimation is inefficient, though consistent (Engle and Sheppard 2001).

Begin with:

$$r_t \mid I_{t-1} \sim N(0, H_t) \tag{1}$$

and: $H_t = D_t R_t D_t, \; where\, D_t = diag\{\sqrt{h_{i,t}}\}$. (2)

In (1) r_t is a *kx1* vector of variable values conditional on information available at *t-1* (I_{t-1}).

R_t is assumed to be conditionally multivariate normal.

The key here is that R_t is a correlation matrix that varies over time, distinguishing the model from the constant conditional correlation model which uses $D_t R D_t$.

H_t is the conditional covariance matrix. D_t is a diagonal matrix of time varying standard deviations, which are obtained form the univariate GARCH specification.

$\sqrt{h_{it}}$ is the conditional standard deviation and the elements of R_t are defined as:

$$\rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{ii,t}q_{jj,t}}}. \qquad (3)$$

The likelihood of the DCC estimator may be written as:

$$L = -0.5\sum_{t=1}^{T}(k\log(2\pi) + 2\log(|D_t|) + \log(|R_t|) + \varepsilon_t^{'}R_t^{-1}\varepsilon_t) \qquad (4)$$

$$= -0.5\sum_{t=1}^{T}(k\log(2\pi) + 2\log(|D_t|) + r_t{'}D_t^{-1}D_t^{-1}r_t - \varepsilon_t^{'}\varepsilon_t + \log|R_t| + \varepsilon_t^{'}R_t^{-1}\varepsilon_t) \qquad (5)$$

Two components in the likelihood function that can vary, the volatility component containing D_t and correlation component containing R_t.

This allows the estimation to be in two parts.

In the first step the volatility term is maximized:

$$L_v = -0.5\sum_{t=1}^{T}(k\log(2\pi) + \log(|D_t|^2) + r_t{'}D_t^{-2}r_t \qquad (6)$$

The next step takes the maximizing value, and the correlation part is maximized:

$$L_c = -0.5\sum_{t=1}^{T}(\log(|R_t| + \varepsilon_t^{'}R_t^{-1}\varepsilon_t - \varepsilon_t^{'}\varepsilon_t). \qquad (7)$$

The DCC parameters are now estimated by using the original likelihood conditional on the first stage univariate parameter estimates.

The dynamic correlations are constructed as:

$$R_t = (1-\alpha-\beta)\overline{R} + \alpha\varepsilon_{t-1}\varepsilon_{t-1}^{'} + \beta R_{t-1} \qquad (8)$$

where α and β are key scalar parameters that are estimated. If $\alpha = \beta$ =0, then R_t is simply $\overline{R}$, and the constant conditional correlation model is sufficient.

Engle (2002) and Tse and Tsui's (2002) DCC GARCH models have GARCH-type dynamics for the conditional correlations and the conditional variances. The advantage of the model can be summed up as, "the model preserves the simple interpretation of the univariate GARCH models, while providing a consistent estimate of the correlation matrix" (Kerney and Poti 2003, 5).

Example of Tse Tests:

Table 1: Tse (2000) Tests for Constant Correlations with Presidential Approval

Economic Variable	Average Correlation	*p* Significance Level*
ICS	0.107	0.017
5-Year Nat. Prospections	0.142	0.003
National Retrospections	0.097	0.056
Personal Prospections	0.132	0.008
Personal Retrospections	0.019	0.468

* One tailed tests with a null of constant correlations.

Example of DCC results:

Table 2: GARCH-DCC(1-1) Estimates for ICS and Approval, 1978-2004

Parameter	Estimate	Standard Error	*t*-value
c_{ICS}	15.862	12.497	1.269
a_{ICS}	0.054	0.077	0.706
$b_{1,ICS}$	0.025	0.757	0.033
$b_{2,ICS}$	-0.230	0.327	-0.703
m_{ICS}	0.041	0.122	0.337
$c_{Approval}$	1.516	0.659	2.300*
$a_{Approval}$	0.423	0.042	10.149***
$b_{1,Approval}$	0.272	0.134	2.036*
$b_{2,Approval}$	0.421	0.125	3.374***
$m_{Approval}$	-0.305	0.062	-4.882***
α	0.058	0.043	1.36
β	0.839	0.104	8.077***
$\overline{R}$		0.107	

* *p*<.05, ***p*<.01,****p*<.001. Estimation is based on the DCC-GARCH model:
$h_t = c_i + a_i u_{t-1}^2 + b_i h_{t-1} + b_i h_{t-2} + m_i u_{t-1}^2 I_{u>0}$ for all i=1,2, and $R_t = (1-\alpha-\beta)\overline{R} + \alpha\varepsilon_{t-1}\varepsilon_{t-1}^{'} + \beta R_{t-1}$.

Figure 2: The Varying Efficacy of National Prospections, 1978-2004