

**Week 10 - RATS Lecture – ARCH Models**  
**March 29, 2006**

Estimating an ARCH model involves using maximum likelihood estimation.

Instead of doing:

```
linreg cons / consr
# constant pmsat
```

Here is an example of a maximum likelihood regression with no ARCH model:

```
nonlin b0 b1 var
frml e = cons - b0 - b1*pmsat
frml LA = -0.5*(log(var) + e(t)**2/var)
compute b0 = 1.0, b1 = .5, var=3.0
maximize(method=bfgs,trace,iterations=200) LA 1 154
```

Which breaks down into:

```
nonlin b0 b1 var
    “nonlin” sets up a list of parameters in the model
    we name 3 parameters to estimate here
frml e = cons - b0 - b1*pmsat
    “frml” is for a ml formula
    “e” is for error.
frml LA = -0.5*(log(var) + e(t)**2/var)
    this sets up the likelihood function – OLS translated into log likelihood
compute b0 = 1.0, b1 = .5, var=3.0
    starting value guesses based on OLS. Var = 3.0 comes from the squared
    Standard Error of Estimate.
maximize(method=bfgs,trace,iterations=200) LA 1 154
    maximize the likelihood function LA for all 154 timepoints.
```

We can put the linreg commands into the program to get exact starting values.

```
nonlin b0 b1 var
frml e = cons - b0 - b1*pmsat
frml LA = -0.5*(log(var) + e(t)**2/var)
linreg cons
# constant pmsat
com b0=%beta(1),b1=%beta(2),var=%seesq,a1=0.05
maximize(method=bfgs,trace,iterations=200) LA 1 154
```

## Testing for ARCH

We want to examine the squared residuals.

```
set ressq = consr*consr
    create new series of squared residuals
correlate(number=20,qstats,dfc=2) ressq
    look at the autocorrelation in the series of squared residuals
@bjident ressq
    look at the ACF and PACF of the series of squared residuals.
```

Lagrange Multiplier Test:

```
linreg ressq
# constant ressq{1}
    regress squared residuals on itself at various lags – here the ACF suggests just
    the first lag.
compute trsq = %nobs*%rsquared
    compute the test statistic.
cdf chisqr trsq 1
    Is the test statistic significant – 1 is for 1 degree of freedom since p=1
```

Now, estimate the ARCH model:

```
nonlin c1 c2 a0 a1
frml ex = cons - c1 -c2*pmsat
frml h = a0 + a1*ex(t-1)**2
frml L = -0.5*(log(h) + ex(t)**2/h)
compute c1 = 15.0, c2 = 0.4, a0 = 8.0, a1 = 0.1
maximize(method=bfgs,trace,iterations=200) L 2 154
```

Line by line:

```
nonlin c1 c2 a0 a1
    give names to parameters to be estimated
    c1 and c2 are parameters for the regression. a0 and a1 are parameters for ARCH
frml ex = cons - c1 -c2*pmsat
    equation for errors of main equation
frml h = a0 + a1*ex(t-1)**2
    this specifies the ARCH process. a1 is the ARCH parameter. We are multiplying
    it by the squared error lagged back one period, that is, its an ARCH-1.
frml L = -0.5*(log(h) + ex(t)**2/h)
    sets up the likelihood function
compute c1 = 15.0, c2 = 0.4, a0 = 1.0, a1 = 0.1
    starting values
maximize(method=bfgs,trace,iterations=200) L 2 154
```

*maximize the likelihood function L. for observations 2-154. We begin with two since we lost one by having a lag of the squared error. default is 20 iterations.*

*“trace” shows us the iterations.*

*“method=bfgs” chooses the Broyden, Fletcher, Goldfarb and Shanno method (described in Press, et al. 1988). This is one of several MLE methods.*

*bhhh is another option that converges more quickly.*

Two other ways to build GARCH models are using the GARCHN.SRC procedure file or using the ARCH/GARCH Wizard in the menu of RATS.

The source file is the older method.

These are ok for simple univariate models but not great for complex models.