SUNY-Stony Brook. Economics Department
Economics 323: Fall 2011
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Problem Set 2 Suggested Solutions.

1. It is not a lie if you believe it! But do you? (Again, justify your answers). (30%)

   a) In the framework of valuing an option to delay an investment, if the average expected price remains the same, but the probability of facing a low price next period becomes smaller, then the NPV of waiting to invest next period will be higher. **True or False**

   False.

   The probability of facing a low price next period becomes smaller the probability of facing a high price next period becomes bigger, and the high price faced next period will be lower (because the expected price remains the same) the recalculated NPV with the new future price will be lower the NPV of waiting to invest next period will be <u>lower.</u>

   b) Any rational investor always prefers a project that gives him, or her, the option to wait to invest next year instead of a project that offers a now or never choice. **True or False**

   True.

   A rational investor prefers to have the option to wait and invest next period if the price goes up, so that he can compare the NPV of waiting and the NPV of investing now and then make a rational decision.

   c) If the uncertainty over prices next period increases, that is, the gap between the possible high price and the possible low price next period increases, keeping the average expected price constant, waiting to invest next period becomes more valuable. **True or False**

   True.

   The gap between the possible high price and the possible low price increases the possible high price next period will be even higher (because we keep the expected price constant and the probabilities of the different outcomes do not change) the NPV with the higher future price will be higher waiting to invest next period become more valuable.

   d) If the stock price at expiration is greater than the exercise price, the value of the call option at expiration will be zero. **True or False**

   False.

If the stock price at expiration is greater than the exercise price (strike price), the value of the call option at expiration will be positive, which is equal to the difference between the stock price and the exercise price.

e) The higher the stock price is at expiration, the lower the value of the put option will be at expiration. **True or False**

True.
For a put option, if the stock price at expiration is greater than the exercise price, the value of the put option at expiration will be zero; if the stock price at expiration is less than exercise price, the value of the put option at expiration will be equal to "the exercise price – the stock price".

2. (40%) A firm is considering investing in a digital camera factory. The factory can be built instantly at a cost *I*, and can produce one camera forever, with no operating costs. The investment is assumed to be irreversible. Currently the price of the camera is $400 but it will change. With a given probability *q* the price will rise to $500, and with probability (1-*q*) will fall to $300. After that the price will remain at that level, higher or lower, forever. We assume that the risk over the future price of camera is unrelated to the rest of the economy, so cash flows can be discounted using the risk-free rate of interest. Take the interest rate to be 10%. Set *I* = $4000, and *q* = 0.5.

a) Given those above values, should the firm invest now, or would it be better to wait a year and see how the price of cameras changes? How much is it worth to have the flexibility to make the investment decision next year, rather than having to invest either now or never?

The expected price is 0.5*500 + 0.5*300 = $400
The NPV of investing now is

$$NPV_1 = -4000 + \sum_{t=0}^{\infty} \frac{400}{(1.1)^t} = -4000 + 4400 = \$400$$

The NPV of waiting until next year is

$$NPV_2 = (0.5)*[\frac{-4000}{1.1} + \sum_{t=1}^{\infty} \frac{500}{(1.1)^t}] = \$681.82$$

The firm should <u>wait</u> until next year.
Compare the results of the above two NPV calculations, the firm should be willing to pay $281.82 (681.82 – 400 = $281.82) more for an investment opportunity that is flexible that one that only allows to invest now.

b) If we think of this in another way, how high an investment cost *I* would the firm be willing to accept to have a flexible investment opportunity rather than an inflexible "now or never" one?

To answer this we find the value of the investment *I* that makes the NPV of the project when we wait equal to the NPV when we invest now, that is, $400.

$$NPV = (0.5)*[\frac{-I}{1.1} + \sum_{t=1}^{\infty}\frac{500}{(1.1)^t}] = \$400$$

We solve the above equation for *I* and get
***I* = $4620**
This means that the opportunity to build the factory now at a cost of $4000 has the same value as an opportunity to build the factory next year at a cost of $4620.

c) If the probability of change in price changes to q = 0.80, and the high price is now $425, and the low price is still $300, should the firm invest now, or would it be a better option to wait a year? Interpret your results.

In this case the probabilities of changes in prices change. But the expected price is still $400. (0.8*425 + 0.2*300 = $400)
The NPV of investing today is still $400. However, the NPV of waiting a year and investing if the price goes up has changed to

$$NPV_3 = (0.8)*[\frac{-4000}{1.1} + \sum_{t=1}^{\infty}\frac{425}{(1.1)^t}] = \$490.91$$

We can see that although waiting is still the best option, the fact that a bad outcome next year is less likely has decreased the value of waiting measured by the difference in NPVs.

d) Now suppose that the uncertainty over price increases. Assume that next period there is a probability q = 0.5 of the price of cameras going up to $600, and the same probability of going down to $200, then would it be better to invest now or to wait? Interpret your results.

In this case, the uncertain over prices increases. The expected price is still $400, but the variability of prices is much higher.
The NPV of investing now is still $400. However, the NPV of waiting a year and investing if the price goes up has changed to

$$NPV_4 = (0.5)*[\frac{-4000}{1.1} + \sum_{t=1}^{\infty}\frac{600}{(1.1)^t}] = \$1181.82$$

We can see that an increase in uncertainty increases the value of waiting, given the larger difference between the NPV of investing now and the NPV of waiting.

e) Based on the original conditions given in the question except that the investment *I* decreases to $2000, should the firm invest now or is it better to wait a year?

The NPV of investing now when I = $2000 has changed to

$$NPV_1 = -2000 + \sum_{t=0}^{\infty}\frac{400}{(1.1)^t} = -2000 + 4400 = \$2400$$

The NPV of waiting and investing if the price goes up has changed to

$$NPV_2 = (0.5)*[\frac{-2000}{1.1} + \sum_{t=1}^{\infty} \frac{500}{(1.1)^t}] + (0.5)*[-2000/1.1 + 300/(1.1)^t] = \$1590.91 + (0.5)*\$1181.82 = \$2181.8$$

Comparing the above two NPVs, it seems like the firm should invest now, which is also clear from the fact that the NPV of the low outcome is also positive.

3. (30%) Suppose you buy a call option of a firm's stock. The current price of the stock is $60, but the price will change next period, with a probability 70% of increasing to $120, and with a probability 30% of decreasing to $30. The exercise price for this call option is $70 at expiration (next period). Assume the interest rate is 5%.

a) What is the present value of your expected profit? (Remember that is the price of the call option in today's dollars)

The expected price is 120*0.7 + 30*0.3 = $93
The expected profit is 93 – 70 = $23
The present value of expected profit is 23/(1.05) = $21.90

b) Now consider a three-period case:

Suppose the stock price in the first period is still $60. In the second period, the stock price will increase to $120 with a probability 70%, and this price will continue to change in the third period, with a probability 40% of increasing to $150 and a probability 60% of decreasing to $100; in the second period, the stock price will go down to $30 with a probability 30%, and this price will also continue to change in the third period, with a probability 40% of going up to $65 and a probability 60% of going down to $20. The call option you buy will expire in the third period and the exercise price is still $70. The interest rate is still assumed to be 5%. Calculate the present value of your expected profit.

The expected price is 150*0.7*0.4 + 100*0.7*0.6 + 65*0.3*0.4 + 20*0.3*0.6 = $95.4
The expected profit is 95.4 – 70 = $25.4
The present value of expected profit is $25.4/(1.05)^2 = \$23.04$