

**SUNY-Stony Brook. Economics Department**  
**Economics 323: Fall 2011**  
**Professor Hugo Benítez-Silva**

Problem Set 1: Suggested solutions. If you have any questions about these answers, or need further clarification, do not hesitate to drop me a line, come to my office hours or the T.A.'s

**1. Let's start kind of easy (10%):**

- a) How much is \$1 Million to be delivered 20 years in the future worth today if the interest rate is 20%?

**Just compute the  $PV = 1M / (1.2^{20})$**

- b) Would the assumption that goods are perfect substitutes be valid in a study of inter-temporal food purchases?

**The answer is no, if we believe people need to consume some food everyday, which is fairly coherent.**

- c) What is the present value of \$100 one year from now if the interest rate is 10%? What is the PV if the interest rate is 5%?

**Follow the rationale in a)**

**2. Truth does matter! (Justify your answers). (10%)**

- a) If the real interest rate is positive, then a unit of future consumption can be had for the sacrifice of less than one unit of current consumption. **True or False**

**True, less than a dollar today buys 1 dollar of consumption tomorrow if the real rate of interest is positive. Notice that if inflation is larger than the nominal rate of interest this might not be necessarily true.**

- b) An increase in the interest rate will necessarily result in a decrease in the present value of a given stream of positive incomes. **True or False.**

**True, nothing too exciting here, just write any PV calculation.**

- c) If apples today are perfect substitutes for bananas today, then apples today must also be perfect substitutes for bananas tomorrow. **True or False.**

**False, here the key is the “must”, preferences over goods today do not imply the same inter-temporal preferences.**

**3. Multiple Choice (10%)**

**A)** Harvey has a utility function  $U(c_1, c_2) = \min\{c_1, c_2\}$  where  $c_1$  and  $c_2$  are his consumption in periods 1 and 2 respectively. Harvey earns \$180 in period 1 and he will earn \$66 in period 2. Harvey can borrow or lend at an interest rate of 10%. There is no inflation. What will Harvey do?

- a) Harvey will save 60
- b) Harvey will borrow 60
- c) Harvey will neither borrow nor lend
- d) Harvey will save 124.5.
- e) None of the above

**This is very similar to the example we had in class, only that now  $m_1 > m_2$ . Since the preferences imply  $c_1 = c_2$ , then we solve for  $c_1$  or  $c_2$  in the inter-temporal budget constraint, for example in present value  $c_1 + c_2/(1+r) = m_1 + m_2/(1+r)$ , and get that  $c_1 = c_2 = 125.714$ , so Harvey will save 54.286.**

**B)** If a consumer views a unit of consumption in period 1 as a perfect substitute for a unit of consumption in period 2 and if the real interest rate is positive, the consumer will:

- a) Consume only in period 1.
- b) Consume only in period 2.
- c) Consume equal amounts in each period.
- d) Consume more in period 1 than in period 2 if income elasticity exceeds 1, else would consume more in period 2 than in period 1.
- e) Equalize expenditures but not consumption in the two periods.

**The answer to question 2a implies that the answer here is b). If you do not care when you consume, you want more, in the future with a positive rate of interest you will have more to consume.**

C) Think of a consumer that only values consumption in periods 1 and 2. She discounts the period 2 consumption by a factor  $\beta < 1$ , and  $\beta < (1/(1+r))$

- a) She will only consume in period 1
- b) She will only consume in period 2
- c) Consume more in period 2 than period 1
- d) None of the above

**The answer is d) since we are told nothing about preferences. If the preferences were to show a high degree of substitutability the right answer would likely be a), but they could be closer to perfect complements, in which case again we would probably choose d).**

#### **4. More Multiple Choices (sometimes using Excel will be convenient here) (30%)**

1. Approximately how many years must one wait for an initial investment of \$10000 to triple in value if the investment earns 9% compounded annually?

- A. 19 B. 17 C. 15 D. 13

**D. 13 years. Some trial and error once you write the formula will get you there.**

2. What is the present value of the following payment stream at an interest rate of 7%; \$1000 today, \$2000 at the end of year 1, \$5000 at the end of year 3, \$6000 at the end of year 5

- A. \$11,243 B. \$10,652 C. \$11,229 D. \$10,034

**C. \$11,229. Notice that the payment today is not discounted.**

3. \$10,000 borrowed at 8% is to be repaid in four equal annual payments. How much of the principal is amortized with the first payment?

- A. \$2481 B. \$2316 C. \$2219 D. \$2187

**C You will have solved it in recitation, so I am hoping you followed that rationale. \$2219 (For this Excel is useful but it is not a must). Notice here that is essential to realize that the total payment in the first period is composed of principal and interest, and that in every period the interest paid are the interest generated on the outstanding balance of the loan at the beginning of that period.**

4. Your 401K's balance is \$50,000 right now. You want to retire in 30 years with \$1,000,000. You add \$6000 annually, but what interest rate do you need to earn in order to reach your goal?

- A. 9.37% B. 8.12% C. 15.76% D. 6.52%

**Here it was easy to get confused, but it was about moving funds from one period to**

**another. You can download a simple spreadsheet with the solution from my page. The exact answer was actually not in the choices, but the minimum interest needed among the choices was B) 8.12%, the right answer is closer to 7.25%, again see the solution.**

5. What is the NPV of a project that costs \$50,000 and returns \$30,000 annually for three years if the opportunity cost of capital is 12%?

A. \$12,047.88 B. \$16,887.97 C. \$22,054.94 D. \$25,374.23

**The answer is C. Notice, however, that if instead of doing it by hand you put it in Excel, this software does not consider the existence of a period 0 for the investment, So the investment happens in period 1, and the flows are in period 2 to 4. That means that the computer NPV in Excel is the discounted value of C, or \$19691.91. If you justified either answer you are fine.**

6. What is the IRR of a project that costs \$74,361.78 and provides cash-flows of \$25,000 annually for four years?

A. 9.85% B. 11.02% C. 13% D. 14.85%

**C. (Using Excel was recommended, without would be really tricky)**

7. If a project requires an initial investment of \$30,000 and returns \$7500 annually for four years, then what can be assumed about the NPV?

A. The NPV is positive; B. The NPV is negative for any interest rate; C. The NPV is zero; D. Nothing can be assumed about the NPV.

**B. If we assume any positive interest rate.**

**C. If we think the interest rate is zero.**

8. A currently used machine costs \$7750 annually to run. What is the maximum that should be paid to replace the machine with one that will last four years and cost \$3575 annually to operate? The opportunity cost of capital is 15%.

A. \$10,000.00 B. \$11,919.53 C. \$11,943.45 D. \$12,348.54

**Here the key is to understand that we are willing to pay as much money as the discounted value of the money we save when operating the machine every year. We save \$4175 from year 1 to year 4, discounting this stream of income at 15% gives us that the answer is B)**

**5. More about Harvey (15%)**

Harvey Habit's utility function is still  $U(c_1, c_2) = \min\{c_1, c_2\}$ , where the arguments are the

consumption in period 1 and consumption in period 2. The price of bread is \$1 per loaf in period 1. The interest rate is 21%. Harvey earns \$2000 in period 1 and he will earn \$1,100 in period 2

- a) Write Harvey's budget constraint in terms of future value, assuming no inflation.
- b) How much bread does Harvey consume in the first period and how much money does he save? (the answer might not be an integer)
- c) Suppose that Harvey's money income in both periods is the same as before, the interest rate is still 21%, but there is a 10% inflation rate. How much would a loaf of bread cost in period 2? Write down Harvey's budget equation for period-1 and period-2 bread, given this new information.

**From Question 3 this should be clear. The only slight difference is in c) where inflation is mentioned. The loaf costs \$1.1 in period 2. The new budget equation in PV terms is**

$$c_1 + (1+\text{inflation rate})/(1+r) c_2 = m_1 + m_2/(1+r)$$

## 6. New Mall (25%)

In recitation you have started to work on the spreadsheet showing the alternative options that New Mall LLP has. Understanding why this problem is useful requires your attention to detail. This is an example of a real investment decision, and real life is complex. In this exercise you will be asked to pay attention to just about every number and be critical about what it means and how you use it, and how using the tools we have talked about in class requires more than just mechanical calculations.

- a) How do the NPV and the IRR calculations change with the interest rate? If interest rates go up, say by 2 percentage points, what happens?

**By the definition of NPV we know that if the interest rate goes up the NPV goes down. If the NPV is still positive, the IRR will go up.**

- b) As we discussed in class and in recitation, almost every time the NPV or IRR approach is used to evaluate projects the assumption of perfect foresight (the analyst knows everything about the future of the project) is used. Imagine for a moment you are presenting this project to a bank officer who will decide whether they give you the \$100k or so, that any of these projects will cost. The loan officer will make a recommendation to the risks committee who will then approve your loan or not.

The main concern of the loan officer is related to the occupancy rates. He wants to know how sensitive your NPV and IRR calculations are to the assumptions regarding occupancy rates, especially of the commercial space.

There are many ways to approach answering the loan officer's concerns. One I suggest is to work out scenarios of different occupancy levels and seeing the effect on NPV and

IRR calculations. Notice, however, that lower occupancy rates not only affect revenues but could also affect costs, since the marketing costs (now buried under miscellaneous) might go up considerably so you can try to rent more space. For example, the current occupancy numbers might be rather optimistic. Can you assign a probability to these numbers and another one to less optimistic numbers? How does the NPV change?

**Here you needed to use a bit your understanding of the problem. You could tell me that the current numbers were optimistic and that represented an scenario that was 80% likely, for example, in the other 20% the occupancy maybe went down by 20%. You had to compute the NPVs in both cases, and maybe change the numbers around, like in the expenses.**

c) Having 72 different tenants can be quite a costly operation, especially because the numbers in the spreadsheet assume yearly contract renovations. On top of that, there is some probability that one of these businesses will face financial hardship, affecting your occupancy rates, or your ability to collect the estimated rent income. In this section you have to read the article I distributed about the big chains of retailers trying to establish themselves around Long Island. Imagine one of these chains (Target, Wal-Mart, Home Depot, etc...) shows interest in the mini-mall you are considering. They essentially offer you to rent the whole property, but they do not want to pay the price per squared meter of commercial real estate that you were planning to charge the small renters. In a traditional NPV calculation that is difficult to reconcile, so you have to be inventive and think about how you can take into account the lower risk and possibly lower costs that renting the whole space to one retailer will have for the business. What would you say the minimum NPV would be to accept the big-box retailer's offer? You have to justify your answer. It is less about the number and more about the reasoning. Can you use the interest rate as a way of considering differences in risks?

**Renting only to one tenant of the quality of Wal-Mart, Target etc. has its advantages, you will spend less in managing the many contracts, and you will get paid for sure. Your occupancy will be 100% if they rent the whole property, and you will get paid with a very high probability, probably 100%. This means a lower rent per squared meter might compensate having the big-box inside. But in order to make the NPV an appropriate tool maybe using the interest rate is coherent here. A stream of income that comes from the big-box retailer might be discounted at a lower interest rate, as a proxy for lower risk, or lower opportunity cost of your resources.**