NPV with an Option to Delay and Uncertainty

- In all the examples and the theory so far, we have assumed perfect foresight when making investment decisions and analyzing future variables.

- We have also ignored that sometimes companies have the option to wait and invest later as a means to reduce the uncertainty they face when making investment decisions.

  Example

- A firm is considering investing in a RP-player factory. The factory can be built instantly at a cost I, and can produce one player forever, with no operating costs. The irreversibility of the investment is assumed.

- Currently the price of the RP-player is \$200 but it will change.

- With a given probability q the price will rise to \$300, and with probability $(1-q)$ will fall to \$100. After that the price will remain at that level, higher or lower, forever.

- We will assume that the risk over the future price of RP-players is unrelated to the rest of the economy, so cash flows can be discounted using the risk-free rate of interest. Take the latter to be 10%.
- Set $I = \$1,600$, and $q = 0.5$.
- Given these values, is this a good investment?
- Should the firm invest now, or would it be better to wait a year and see how the price of RP-players change?

  1. First, use the traditional NPV calculation. Notice that the expected future price of RP-players is always \$200.

     $$NPV_1 = -1600 + \sum_{t=0}^{\infty} \frac{200}{(1.1)^t} = -1600 + 2200 = \$600 \tag{1}$$

     This delivers a positive NPV, so it looks like we would go ahead with the project.

2. However, this traditional calculation ignores the cost of investing now, rather than waiting and keeping open the possibility of not investing if the price falls. So now we recalculate the NPV but assuming the firm waits one year and only invests if the price goes up to $300.

$$\begin{aligned} NPV_2 &= (0.5)\left[\frac{-1600}{1.1} + \sum_{t=1}^{\infty} \frac{300}{(1.1)^t}\right] \\ &= (0.5)\,(-1454.54 + 3000) = \$773 \end{aligned}$$

Clearly it is better to wait than invest right away.

- Notice that for this calculation to make sense two things are needed: irreversibility of the initial investment, and the possibility of waiting for the price to move.

- Here we can ask ourselves, how much is it worth to have the flexibility to make the investment decision next year, rather than having to invest either now or never?

  The answer is simple, just compare the results of the NPV calculations: We should be willing to pay $173 more for an investment opportunity that is flexible than one that only allows us to invest now.

- Another way to look at this is to ask the following question: How high an investment cost I would we be willing to accept to have a flexible investment opportunity rather than an inflexible "now or never" one?

- To answer this we find the value of the investment, which we denote by $\bar{I}$, that makes the NPV of the project when we wait equal to the NPV when $I = \$1,600$ and we invest now, that is, \$600. That means solving:

$$NPV = (0.5)\left[\frac{-\bar{I}}{1.1} + \sum_{t=1}^{\infty} \frac{300}{1.1^t}\right] = \$600. \qquad (2)$$

  If we solve for $\bar{I}$ we find that it is equal to \$1,980. This means that the opportunity to build a RP-player factory now at a cost of \$1,600 has the same value as an opportunity to build a factory now or next year at a cost of \$1,980.

- Now as an extension let us consider the case when the probabilities of changes in prices change.

- Assume now that $q = 0.75$ and that the high price is now \$233.33, and the low price is still \$100. The average expected price is still \$200.

  The NPV of investing today is still \$600, since nothing has changed in expectations terms for an investor today. However the NPV of waiting a year and invest if the price goes up has changed

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\begin{aligned}
NPV_3 &= (0.75)\left[\frac{-1600}{1.1} + \sum_{t=1}^{\infty}\frac{233.33}{(1.1)^t}\right] \\
&= (0.75)\,(-1454.54 + 2333.33) = \$659.1
\end{aligned}
$$

  We can see that although waiting is still the best option, the fact that a bad outcome next period is less likely has decreased the value of waiting measured by the difference in NPVs.

- Finally, another extension we can consider is increasing the Uncertainty over Price.
- The way we consider this is by assuming that next period there is a probability $q = 0.5$ of the price of RP-players going up to \$350, and the same probability of going down to \$50. The expected price is still \$200, but the variability of prices is much higher.

  The NPV of investing today is still \$600. But now we have to recompute the NPV with the new future prices:

$$\begin{aligned} NPV_4 &= (0.5)\left[\frac{-1600}{1.1} + \sum_{t=1}^{\infty} \frac{350}{(1.1)^t}\right] \\ &= (0.5)(-1454.54 + 3500) = \$1,022.72 \end{aligned}$$

  We can see that an increase in uncertainty increases the value of waiting, given the larger difference between the NPV of investing now and the NPV of waiting. This is the case because the higher uncertainty increases the upside potential payoff, but leaves the downside potential unchanged at zero.