

# Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence Zeldes, QJE 1989

## Background (Not in Paper)

- Income Uncertainty dates back to even earlier years, with the seminal work of Beckmann (1959). His is one of the first examples of use of Dynamic Programming in economics. He only deals with simple characterizations of the model, but sets the stage for further work.
- Nagatani (1972) also relaxes the income certainty assumption, and using Isoelastic utilities (which implies precautionary savings, with its risk aversion parameter also controlling the degree of precautionary savings) shows that the allocation of consumption is not independent of the income realizations, even without having to resort to borrowing constraints. Basically, he shows that the consumer is revising his income prospects as the life cycle proceeds, resulting in the positive relationship between consumption and income.
- Miller (1974) provides an infinite horizon model with stochastic income stream, and is a closer counterpart with this type of uncertainty to the work of Phelps, and Hakansson. He finds that this source of uncertainty leads to a lower consumption stream than in the certainty case.
- Instead of explicitly solving the CDP model for the optimal consumption rule  $\delta^*$ , Hall's (1978) test of the life-cycle/permanent income hypothesis was based on the Euler equation to the dynamic problem. Hall's model generalizes Phelps's model by allowing stochastically evolving labor income  $\{y_t\}$ , but assumes that  $R$  is the (non-stochastic) risk-free rate of return on bonds.
- In this case the Euler equation takes the form:

$$0 = u'(c_t) - \beta RE \{u'(c_{t+1}) | H_t\}. \quad (1)$$

Hall observed that (1) implies that  $c_t$  is a “sufficient statistic” for  $c_{t+1}$ . For example, in the special case where  $R = 1/\beta$ , (1) says that  $\{u'(c_t)\}$  is a martingale. Hall tested this by running the regression:

$$c_{t+1} = \alpha_1 + \alpha_2 c_t + \alpha_3 H_t + \xi_t. \quad (2)$$

If  $u$  is quadratic, then it is not difficult to see that (1) implies that the regression in (2) will hold with  $\alpha_3 = 0$ : i.e. no other information in consumers' information set  $H_t$  will be useful predictors of  $c_{t+1}$  once we know  $c_t$ . Using elements  $c_{t-1}$ ,  $y_t$  and  $y_{t-1}$  of  $H_t$  (where  $y_t$  denotes income), Hall was unable to reject the hypothesis that  $\alpha_3 = 0$  using U.S. aggregate time series data from 1948 to 1977.

- Flavin (1981) re-investigated Hall's findings, estimating a structural model with quadratic utility and an explicit AR representation for the stochastic process of income  $\{y_t\}$ :

$$y_t = \rho_0 + \rho_1 y_{t-1} + \dots + \rho_n y_{t-n} + \varepsilon_t, \quad (3)$$

where  $\{\varepsilon_t\}$  is a white-noise disturbance process. Flavin showed that disturbance  $\varepsilon_t$  is proportional to the consumers' re-evaluation of their permanent income at time  $t$ . If the permanent income hypothesis holds, we have:

$$\begin{aligned} \Delta c_t &\equiv c_t - c_{t-1} = k \varepsilon_t \\ k &= \frac{(R-1)}{1 - \rho_1 R^{-1} - \rho_2 R^{-2} - \dots - \rho_n R^{-n}} \end{aligned} \quad (4)$$

Thus, (4) says that changes in consumption are driven entirely by "surprises" in labor income which alter consumers' view of their permanent income. Flavin tested the permanent income hypothesis by estimating the income process (3) together with the regression:

$$\Delta c_t = k(y_t - \rho_0 - \rho_1 y_{t-1} - \dots - \rho_n y_{t-n}) + \phi_0 + \phi_1 \Delta y_{t-1} + \dots + \phi_m \Delta y_{t-m} + u_t, \quad (5)$$

testing the hypothesis that the coefficients on lagged income changes  $(\phi_0, \dots, \phi_m)$  are zero. Flavin found that the estimated  $\phi$ 's were significantly different from zero, leading her to conclude

"The empirical results indicate that the observed sensitivity of consumption to current income is greater than is warranted by the permanent income-life cycle hypothesis, even when the role of current income in signalling changes in permanent income is taken into account. The restrictions implied by the permanent income-rational expectations hypothesis can be rejected statistically at very high confidence levels. Further, the estimates of the marginal propensity to consume out of current income are quite large, indicating that the failure of the permanent income hypothesis is quantitatively, as well as statistically, significant." (p. 976-977).

- Flavin's finding that consumption responds to predictable innovations in income has since become known as the problem of *excess sensitivity*, and was confirmed in subsequent studies including Hall and Mishkin (1982) using the PSID. In his survey of the literature Deaton (1985) concluded: "Surprise consumption functions estimated on quarterly U.S. aggregate time series show clear evidence of excess sensitivity of consumption to predictable events." (p. 146).
- However recent research has questioned whether there really is an excess sensitivity puzzle. Mariger and Shaw's (1988) reinvestigation of the Hall-Mishkin results suggests that the excess sensitivity might be a phenomenon of the early 1970's, since they find little evidence of excess sensitivity in subsequent waves of the PSID. Zeldes (1989) points out that the excess sensitivity finding may be a rejection of the certainty equivalence principle implied by the quadratic utility specification used by Hall, Flavin, and Deaton. Zeldes numerically computed the optimal consumption rule  $\delta^*$  in a model with CRR preferences and concluded:
 

"The results indicate that rational individuals with constant relative risk aversion utility will optimally exhibit "excess" sensitivity to transitory income, save "too" much, and have expected growths of consumption that are "too" high, relative to the simple permanent income hypothesis benchmark, even in the absence of borrowing constraints. This suggests that we should rethink our presumption that the certainty equivalent model is the appropriate benchmark, especially at low levels of financial wealth. (p. 295-296).
- Attanasio and Browning (1991) argue that the rejections of the life-cycle hypothesis in models estimated using aggregate time series data "may plausibly be attributed to aggregation bias." (p. 2). Using detailed micro data from the U.K. Family Expenditure Survey, they find that "although very simple forms of the consumption function seem to display an excess sensitivity to expected change in income this completely disappears when we allow for the effects of demographics and labor supply." (p. 3-4). Runkle (1991) obtains similar results using the PSID data, concluding: "Aggregate studies assume a representative agent who conditions expectations on *aggregate* variables. But households may not find aggregate data useful in predicting their future economic conditions. In this study I assumed only that each household knew its own past economic condition. With this assumption, I could not reject the permanent income hypothesis." (p. 91).

- Another possible reason for the excess sensitivity finding is measurement error. Altonji and Siow (1986) and Runkle (1991) found significant measurement error in income and consumption variables in the PSID. They re-estimated (2) using an instrumental variables estimator which is consistent in the presence of measurement error. Contrary to the findings of Hall and Mishkin (1982), they found that no variables in  $H_t$  were significant (not even lagged income changes).
- Skinner (1988) uses an approximation to the optimal decision rules to show how important precautionary savings are in the presence of uncertain income streams. With convex marginal utility of consumption we will see precautionary savings, to insure against the uncertain future path of income.
- Precautionary savings models will predict that people save a lot more when young than what a simple life cycle model would predict. Also some work shows that lack of borrowing can be the result of a rational response to the uncertainty of the income stream that could lead to zero consumption.

## Motivation

- No one has derived closed-form solutions for consumption with stochastic labor income and constant relative risk aversion utility.
- This means relaxing the ability to resort to the certainty equivalence (CE) result that applies when using quadratic utility and it is allowed to have negative consumption.
- The traditional result is that expected consumption is constant over time. Consumption is proportional to the expected present value of lifetime resources, which consist of human wealth, plus non-human wealth. Human wealth is the present discounted value as of time  $t$  of expected future labor income.
- The author finds substantial differences between the simplified CE model and the optimal consumption functions computed numerically, and potentially explain the high sensitivity of consumption to transitory income, the high growth of consumption in the presence of low risk-free interest rate, and underspending of the elderly.
- With his models the author can compute the optimal amount of precautionary savings, and calculate the optimal sensitivity of consumption to both permanent and transitory changes in income.
- He allows for lending and borrowing at a riskless rate, with only a no die in debt condition.

## The Puzzles

- Consumption seems to be too sensitive to changes in current income. Hall and Mishkin (1982) define excess sensitivity as the difference between the response in consumption and the annuity value of the increase in human and nonhuman wealth that occurs as a result of this innovation. They use the PSID and find this excess sensitivity.
- Expected growth of consumption over time in a certainty model with time separable utility and an interest rate less than the rate of time preference is expected to be negative. This is not the case empirically as reported by Deaton (1986) and Singleton (1985).

- The elderly do not seem to dissave sufficiently fast if we were to believe the benchmark life cycle model with certainty and no bequest . Some authors have found they do dissave, others that not at all.
- Are these puzzles real puzzles, considering the very simple benchmark model. Are agents not rational, not forward looking, or maybe everything is explain by borrowing constraints.

## The Model

- Individual lives many periods, chooses optimal current consumption and contingency plans for future consumption to maximize the expected value of a lifetime time-separable utility function.
- The only source of uncertainty is exogenous future labor income, and individuals cannot hedge against this by trading contingent claims.
- The consumer chooses  $C_t$  to maximize

$$E_t \sum_{j=0}^{T-t} \left( \frac{1}{1+\delta} \right)^j U(C_{t+j}), \quad (6)$$

subject to

$$W_{t+1} = (W_t - C_t)(1 + r_t) + Y_{t+1} \quad (7)$$

$$C_t \geq 0 \quad (8)$$

$$W_T - C_T \geq 0. \quad (9)$$

- Future value at time  $T$  of initial wealth and subsequent realizations of income must be at least as great as the future value of the chosen consumption path: the individual must pay back all loans with probability one before the end of life.
- The author assumes that the interest rate is constant and equal to the rate of time preference.
- For any concave utility function with non-stochastic income the solution is that consumption is proportional to the expected present value of lifetime resources.

- This is not true in general if the income is stochastic unless utility is quadratic and consumption is allowed to take negative values. This is the Certainty-Equivalence solution.
- In terms of the sensitivity of consumption with respect to current income, CEQ predicts that consumers respond in the same way to an extra dollar of income as to an extra dollar of wealth. This will not be the case here.

## Implementation

- The model is calibrated based on panel data estimates of the amount of income uncertainty facing individuals in the economy. He divides income into a permanent component, and a transitory component. He uses some ad-hoc probability distribution over the evolution of these components.
- The method for calculating the optimal consumption is stochastic dynamic programming. One state variable (wealth), one control variable (consumption), and one-disturbance (income). He discretizes the state space to approximate the solution.
- To adequately take into account the evolution of the permanent income component he needs to introduce another state variable, making the solution of the model more challenging.

## Results

- For simplicity he assumes a 15 period model.  $\gamma$  is equal to 3. The model exhibits dramatic “excess sensitivity” relative to the CE benchmark.
- His results indicate that we should expect MPCs out of transitory income to be much larger for households with low current (certain) assets relative to expected future (uncertain) income than for the rest of the population.
- He also finds that individuals “overrespond” to changes in current income or wealth, and “underrespond” to changes in expected future income.
- Then defining precautionary savings as  $C - C_{CEQ}$  calculates that consumption would be 20% higher if there were no uncertainty in the model. Conjectures that this means that a lot of the wealth accumulation we see in the US is due to precautionary savings.

- He also finds that consumption growth is positive even if the interest rate is equal to the rate of time preference.
- In terms of the *dissaving when old* “puzzle” he finds that if we interpret possible catastrophic health expenditures events as possible negative income shocks, the model would predict that precautionary savings would lead to less dissaving than the standard model would predict.

## Open Questions

- If precautionary savings are important you would expect individuals to accumulate substantial wealth. Is this true?
- Do individuals that face greater uncertainty accumulate more savings? Still unresolved. Controlling for financial assets, income profiles, and risk aversion is not easy.
- He correctly conjectures that endogenizing labor supply would provide another way of hedging the income uncertainty. He sees that the effect of uncertainty on consumption might be less dramatic. Obviously it becomes about smoothing utility.