The Neoclassical Labor Supply Model: Intertemporal and Life Cycle Models

Economics 640

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1 Basic Setup

The static model of labor supply discussed so far is quite restrictive. A more realistic model of labor supply and participation behavior will need to recognize that individuals make decisions in multiple periods. The life cycle model of labor supply assumes that individuals maximize the rest-of-their-life utility subject to constraints. This inter-temporal framework generalizes the scope of labor supply and participation research in several important ways compared to the static model:

- Variations in labor supply and participation behavior over the individual's life cycle, which include retirement behavior, can be explored.
- Behavioral responses in period and total life-time labor supply, to changes in the wage in a single period or to changes in the entire wage profile can be analyzed.
- Differences in savings and borrowing behavior across individuals and over the life cycle can be explained.
- The exogenous and endogenous nature of non-labor income can be explored since initial assets are given and wealth can accumulate over time.
- The effect of stochastic events over the life course such as wage uncertainty, interest rate uncertainty, health uncertainty, or unemployment uncertainty, on consumption, wealth accumulation, and labor supply behavior can be investigated.
- The role of expectations and their formation can be analyzed.

[1] Frank Heiland authored an early version of these notes. All remaining errors, however, are my own.

As in the static model, we assume that the individual derives utility in a given period from consuming a market-purchased consumption good, $C(t)$, and time spend outside of the labor market ('leisure'), $L(t)$. The individual's preferences are summarized by the following strictly concave period utility function,

$$U(t) = U(C(t), L(t); X(t), \varepsilon^*(t)) \tag{1}$$

where the $X(t)$ is a vector of period-specific observable characteristics of the individual and $\varepsilon^*(t)$ is a scalar representing unobserved characteristics or tastes that affect her period utility. The additively separable intertemporal utility function of an individual who knows that her life ends after $K+1$ periods is:

$$U = \sum_{t=0}^{K} \left(\frac{1}{1+\rho}\right)^t \cdot U(C(t), L(t); X(t), \varepsilon^*(t)), \tag{2}$$

where $\rho \geq 0$ is the subjective time preference rate of the individual.

Additive separability implies that (1) the marginal rate of substitution between leisure and consumption in period t is independent of the amounts of leisure and consumption enjoyed in all other periods (weak separability), and that (2) the marginal rate of substitution between leisure (consumption) in period t and leisure (consumption) in period $t+1$ is independent of the amounts of leisure and consumption enjoyed in all periods other than t and $t+1$ (strong separability). This assumption is problematic particularly if the length of the periods is short.[2]

At the beginning of her life the individual has an asset endowment of $A(0)$, which will be assumed to be exogenous. It is possible to account for initial conditions econometrically, and at least in part through more detailed constructions of the utility function to include bequest motives, for example.

Lifetime utility is maximized subject to a lifetime wealth constraint:[3]

$$A(0) + \sum_{t=0}^{K} \left(\frac{1}{1+r}\right)^t \cdot W(t) \cdot H(t) = \sum_{t=0}^{K} \left(\frac{1}{1+r}\right)^t \cdot P(t) \cdot C(t), \tag{3}$$

where $W(t)$ is the exogenously given wage profile, $H(t) = T - L(t)$, represents the labor supply profile, $P(t)$ stands for the price of the consumption good, and r is the real interest rate in the capital market at which the individual can borrow or save.

[2] A critical review of the life cycle approach can be found in Frederick et al. (2002).

[3] Incorporating uncertainty with regard to wages (and interest rates) in the model is relatively straightforward (see e.g., Altonji (1986), or Card (1994)) and will be discussed below.

The individual chooses the consumption path, $C(t) > 0$, and hours worked, $H(t) \geq 0$, to maximize her utility, expression (2), subject to the wealth constraint, expression (3).

Kuhn-Tucker conditions for an optimum are the satisfaction of the lifetime budget constraint and

$$\frac{\partial U}{\partial C(t)} = (\frac{1+\rho}{1+r})^t \cdot \lambda \cdot P(t), \ \ t = 0, ..., K, \tag{4}$$

$$\frac{\partial U}{\partial L(t)} \geq (\frac{1+\rho}{1+r})^t \cdot \lambda \cdot W(t), \ \ t = 0, ..., K, \tag{5}$$

where λ is the Lagrange multiplier associated with the wealth constraint. Along the optimal path, it represents the marginal utility of initial wealth. It can be seen that along the optimal consumption path the marginal utility of consumption equals the price of the consumption good adjusting for the marginal utility of wealth, time preference, and interest rate.

Expression (5) characterizes participation and hours worked behavior over the life cycle. In a given period (a) all time is spend on leisure if the marginal utility of leisure time at full leisure (i.e. H(t)=0) exceeds the discounted period wage rate adjusted for the marginal utility of wealth, or (b) time is spend in the labor market, $H(t) > 0$, if the marginal utility of leisure at $H(t) > 0$ equals the discounted period wage rate adjusted for the marginal utility of wealth.

More formally, the optimal consumption and labor supply functions depend on λ, $W(t)/P(t)$, and $\theta^t = (\frac{1+\rho}{1+r})^t$ as follows:

$$C(t) = C(\theta^t \cdot \lambda, W(t), P(t); X(t), \varepsilon^*(t)), \ \ t = 0, ..., K, \tag{6}$$

$$H(t) = H(\theta^t \cdot \lambda, W(t), P(t); X(t), \varepsilon^*(t)), \ \ t = 0, ..., K \tag{7}$$

$$= \begin{cases} H(t) > 0 & \text{if } \frac{\frac{\partial U}{\partial L(t)}}{\lambda} = -\frac{\frac{\partial U}{\partial H(t)}}{\lambda} = \theta^t W(t) \\ H(t) = 0 & \text{if } \frac{\frac{\partial U}{\partial L(t)}|_{H(t)=0}}{\lambda} = -\frac{\frac{\partial U}{\partial H(t)}|_{H(t)=0}}{\lambda} \geq \theta^t W(t), \end{cases}$$

Similar to the static model of labor supply we can define

$$\frac{\frac{\partial U}{\partial L(t)}|_{H(t)=0}}{\lambda} = -\frac{\frac{\partial U}{\partial H(t)}|_{H(t)=0}}{\lambda}$$

as the reservation wage.

Notice that unlike in the static model, the participation and hours worked decision in a period not only depends on the current wage but also on the constant λ, the marginal utility of wealth, that summarizes all life cycle price and taste information. The effects of λ and $W(t)$ on life cycle behavior can be summarized in the following proposition.

Inspecting conditions (4)-(5), one finds that (strict) concavity of $U(.)$ and normality of consumption and leisure imply that,

$$\frac{\partial C(t)}{\partial \lambda} < 0, \frac{\partial C(t)}{\partial \theta^t} < 0, \frac{\partial H(t)}{\partial \lambda} \geq 0, \frac{\partial H(t)}{\partial \theta^t} \geq 0, \frac{\partial H(t)}{\partial W(t)} \geq 0, \quad (8)$$

where the three expressions on hours worked hold with strict inequality for the interior solution.[4]

The predictions are partial derivatives of the consumption demand and labor supply equation, i.e. in particular the derivatives assume that λ is held constant. The third and fifth expression are particularly important in the what follows.

The life cycle model predicts a positive association between period wage and period hours supplied holding the marginal utility of wealth, i.e. λ, constant. Intuitively, participation is more likely and labor supply is greater when the opportunity cost of leisure increases holding the marginal utility of wealth constant. Below we will explore this intertemporal substitution effect of hours worked (or leisure) and how it is related to familiar comparative static wage effects.

To further explore λ, we evaluate the wealth constraint at the optimum,

$$A(0) + \sum_{t=0}^{K} (\frac{1}{1+r})^t \cdot W(t) \cdot H(\theta^t \cdot \lambda, W(t), P(t); X(t), \varepsilon^*(t)) = \quad (10)$$

$$\sum_{t=0}^{K} (\frac{1}{1+r})^t \cdot P(t) \cdot C(\theta^t \cdot \lambda, W(t), P(t); X(t), \varepsilon^*(t)),$$

[4] Proof (Heckman 1974): Let $A = \theta^t \lambda W(t)$ and $B = \theta^t \lambda P(t)$. The leisure and consumption demand function for the interior solution are then $L(t) = L(A,B)$ and $C(t) = C(A,B)$. Using this notation the FOCs, (4)-(5), are $\frac{\partial U}{\partial L}[L(A,B), C(A,B)] = A$, and $\frac{\partial U}{\partial C}[L(A,B), C(A,B)] = B$. Total differentiation of these equations with respect to A and B yields (in matrix notation):

$$\begin{pmatrix} U_{LL} & U_{LC} \\ U_{CL} & U_{CC} \end{pmatrix} \times \begin{pmatrix} L_A & L_B \\ C_A & C_B \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (9)$$

where subscripts denote partial derivatives. The solution of the system is:

$$\begin{pmatrix} L_A & L_B \\ C_A & C_B \end{pmatrix} = 1/|K| \cdot \begin{pmatrix} U_{CC} & -U_{LC} \\ -U_{CL} & U_{LL} \end{pmatrix},$$

where $|K| = \begin{vmatrix} U_{LL} & U_{LC} \\ U_{CL} & U_{CC} \end{vmatrix} > 0$ (by strict concavity of the period utility function). Hence, $L_A = \frac{\partial L}{\partial A} > 0$, $C_B = \frac{\partial C}{\partial B} > 0$, and $L_B = C_A$. Of course this implies $\frac{\partial H}{\partial A} \leq 0$, with equality if no labor is supplied in the period. The propositions follow by the definition of A and B. Q.E.D.

which implicitly determines λ as a function of initial assets, lifetime wages, lifetime prices, interest rate, time preference rate, and individual's characteristics and tastes, i.e.

$$\lambda = \lambda(W(0), W(1), ..., W(K); P(0), ..., P(K); A(0); \rho; r; X(0), ..., X(K); \varepsilon^*(0), ..., \varepsilon^*(K)).$$

Inspecting the equation that defines λ, (10), and noting that the λ-constant function $H(t)$ ($C(t)$) above is strictly increasing (decreasing) in λ for an interior solution (compare proposition (8)) it must be that

$$\frac{\partial\lambda}{\partial A(0)} < 0, \tag{11}$$

and by the additional assumption that consumption and leisure are normal goods one can show that[5]

$$\frac{\partial\lambda}{\partial W(t)} \leq 0, \;\; t = 0, ..., K.$$

The proposition says that the marginal utility of wealth (based on initial assets) diminishes with wages and initial assets. It is key to note that λ is a permanent effect much like permanent income in the well-known life cycle model of consumption (see e.g., Heckman (1974)) since it does not change over an individual's life cycle. Technically speaking it is a sufficient statistic for all historic and future information. In the absence of wage uncertainty it does not need to be revised after the first period since the wage and price paths are known from the initial period on by assumption. We can analyze how different levels of λ affect life cycle behavior based on differences in the wage profiles and tastes (etc.) between individuals (or for the same otherwise identical individual) that imply different levels of λ.

[5] The Proofs are left as an exercise that you should try.

2 Empirical Implications

The standard life cycle model of labor supply described above has been implemented empirically for men in MaCurdy (1981) [6] and for women in Heckman and MaCurdy (1980, 1982). We will begin the discussion with the implementation of the male labor supply model which is simpler than the female labor supply model since we can abstract from the participation decision and focus on hours worked.

2.1 Male Labor Supply

MaCurdy (1981) specifies the following additively separable (addilog) utility function for individual i, $i = 1, ..., N$:

$$U_i(C(t), H(t)) = \gamma_{C_{i,t}} \cdot C^{\beta}(t) - \gamma_{H_{i,t}} \cdot H^{\alpha}(t), \tag{12}$$

where $0 < \beta < 1 < \alpha$ for concavity. Notice that this is additive over time, and also additive in consumption and hours within any given period. Unobserved and observed heterogeneity are introduced through

$$\ln \gamma_{H_{i,t}} = X_i - \varepsilon^*_{i,t}$$

where $\varepsilon^*_{i,t}$ are i.i.d. mean zero random variables and X_i represents constant individual-specific characteristics.

Assuming an interior solution path, the λ-constant labor supply function is

$$\ln H_{i,t} = F_i + b \cdot t + \delta \cdot \ln W_{i,t} + \varepsilon_{i,t}, \tag{13}$$

with

$$F_i = \frac{1}{\alpha - 1} \cdot (\ln \lambda_i - X_i - ln\alpha), \tag{14}$$

and

$$\delta = \frac{1}{\alpha - 1}, b = \delta \cdot (\rho - r), \varepsilon_{i,t} = -\delta \cdot \varepsilon^*_{i,t}, \rho - r \approx \ln \frac{1 + \rho}{1 + r}.$$

MaCurdy's specification results in a linear panel model.

[6] The first attempt to estimate the model is Becker and Ghez (1975). More recently Antonji (1986) improves upon MaCurdy's empirical specification.

The individual-specific effect F_i has to be treated as a fixed effect to avoid biased estimates since it is correlated with $W_{i,t}$ (or the instruments used to identify the period wage effect) via λ. Equation (13) can be estimated using first differences with panel data or synthetic cohort data. The estimates yield insight in the dynamic labor supply behavior of a particular individual (i.e. holding λ constant). Specifically, $\delta > 0$ captures the response of labor supply to changes of the wage over the life cycle. MaCurdy refers to these changes as *evolutionary changes* or changes along the wage profile as opposed to *parametric changes* or changes of the entire wage profile.

He refers to δ as the intertemporal substitution elasticity of labor supply, i.e. the response (in %) of hours worked to a one percent increase in the wage over the life cycle holding everything else constant (in particular λ).

The identification of determinants of labor supply across individuals is more difficult. Differences in initial assets or wages, for example, affect labor supply through F_i via λ_i but even for relatively simple preferences like those above an implicit expression for λ cannot be obtained (compare expression (10)).

To proceed, MaCurdy assumes that F_i can be approximated by the following linear function

$$F_i = Z_i \cdot \phi + \sum_{t=0}^{K} \gamma(t) \cdot \ln W_i(t) + A_i(0) \cdot \kappa + \mu_i, \tag{15}$$

where Z_i denotes a vector of observed individual characteristics, μ_i is an error term, and ϕ, $\gamma(t)$, and κ are parameters that are assumed to be approximately constant across individuals. From the theoretical predictions in expressions (11) we know that the $\gamma(t)$'s and κ should be negative.

Two issues remain concerning the estimation of equation (15). First, the dependent variable is not observed. Second, data on individuals' wage profiles and initial asset holdings are needed. The first problem is solved by using the predicted fixed effects based on first difference or within-estimates of equation (13). The individual fixed effects are recovered from predicted average age-invariant differences, i.e.

$$\tilde{F}_i = \frac{1}{\tau} \sum_{j=1}^{\tau} (\ln H_{i,j}(t) - \hat{b} \cdot t(j) - \hat{\delta} \cdot W_{i,j}(t)), \tag{16}$$

where τ denotes the number of waves in the (balanced) panel.

MaCurdy addresses the second problem by assuming that the wage profile is quadratic in age of the individual and that the coefficients that determine the shape of the profile, $\pi_{0,i}$,$\pi_{1,i}$, and $\pi_{2,i}$, are linear functions of observed individual-specific characteristics that are constant over the life cycle (MaCurdy uses for example educational attainment)

$$\ln W_i(t) = \pi_{0,i} + t \cdot \pi_{1,i} + t^2 \cdot \pi_{2,i} + \eta_i(t), \tag{17}$$

where $\eta_i(t)$ is an error term. In the paper it is shown how variables $\tilde{\pi}_{0,i}$,$\tilde{\pi_{1,i}}$, and $\tilde{\pi}_{2,i}$ with means equal to $\pi_{0,i}$, $\pi_{1,i}$, and $\pi_{2,i}$ can be constructed based on the life cycle wage data.[7] The procedure to construct initial wealth measures in the absence of a ready-made instrument is detailed in the article. Finally, using $\tilde{F_i}$ as dependent variable and $\tilde{\pi}_{0,i}$, $\tilde{\pi}_{1,i}$, and $\tilde{\pi}_{2,i}$ (and initial wealth) as independent variables, a system of simultaneous equations is formed and the parameters of interest, expression (15), can be identified from the system estimation.

Based on expressions (13), (15), and (17) we now proceed to the analysis of the effect of *parametric wage changes*, i.e. different wage profiles, on labor supply. The investigation applies familiar comparative static analysis to the life cycle allocation problem. The kind of changes of the wage profile that can be analyzed depends on the assumed functional form of the wage profile (see above).

We first consider two wage profiles (in logs) where profile II is higher than profile I during some period s and identical elsewhere. From proposition (11) we know that $\lambda_{II} < \lambda_I$. In other words the reservation wage profile for the person with the higher wage profile (II) will be above the reservation wage profile of person I (compare proposition (8)). Hence, for all periods except period s individual II will unambiguously supply less time to the labor market than individual I. Given the MaCurdy (1981) specification, the effect of a wage change in period s of magnitude $\Delta = \ln(W_{II}(s) - W_I(s)) > 0$ results in $F_{II} - F_I = \gamma(s) \cdot \Delta < 0$. Consequently the life cycle labor supply profile (in logs) for II is approximately lower by a constant compared to I for all periods other than s. In period s, hours worked (in logs) for II differ from I by $(\delta + \gamma(s)) \cdot \Delta$ which may be positive if δ is sufficiently large. We just showed that the elasticity of an evolutionary wage change is larger than the elasticity of a comparable parametric change, i.e. $\delta > \delta + \gamma(s)$. This is of course the result of the life-time wealth change in λ due to the wage effect for the parametric wage change.

[7] To analyze a set of variables in a regression framework it is sufficient to have a set of measures that are means of the variables of interest.

Also, notice that with the utility function at hand, $\gamma(s)$ and $\delta+\gamma(s)$ are approximations of the own- and cross-uncompensated wage elasticities of labor supply (derivative of Marshallian demand function with respect to wage).[8]

Now we turn to a related parametric wage change where the intercept of the wage profile (in logs) shifts upwards, i.e the entire wage profile shifts up. The higher wage profile is denoted by III. In the MaCurdy specification the intercept term is denoted by π_0. From the general proposition (11) we know that $\lambda_{III}(\pi_{III,0}) < \lambda_I(\pi_{I,0})$ where $\pi_{III,0} > \pi_{I,0}$. The reduction in F under profile III increases the reservation wage profile. At the same time the higher wage in each period increases the opportunity cost of leisure in each period. Given the MaCurdy (1981) specification the effect of $\Delta = \pi_{III} - \pi_I > 0$ on F is $F_{III} - F_I = \sum_{t=0}^{K} \gamma(t)\cdot\Delta < 0$. The direct effect on hours worked (in logs) is $\delta\cdot\Delta$. Consequently, the net effect is $(\delta+\sum_{t=0}^{K}\gamma(t))\cdot\Delta$ which may be negative. Also, note that we can expect this parametric wage elasticity to be smaller than the one discussed above, i.e. $\delta+\gamma(s) > \delta+\sum_{t=0}^{K}\gamma(t)$ where s in $\{0,...,K\}$.

MaCurdy reports estimates of the intertemporal elasticity of substitution (using wages), δ, between .1 and .23 from a 10 year panel of 513 white continuously married men, age 25-46 in 1967, from the 1968-77 PSID. The corresponding estimates of effects of a shift in the intercept of the wage profile on F range from $-.05$ to $-.1$. Since these parametric effects are smaller in magnitude than the evolutionary effects, the net effect of an upwards-shift in the wage profile on labor supply is positive, i.e. $\delta+\sum_{t=0}^{K}\gamma(t) > 0$. For additional results and a more detailed discussion see MaCurdy (1981).

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[8] We can also derive the own- and cross-*compensated* wage elasticities. Note that in this model initial assets, $A(0)$, plays the role of non-labor income in the static model. Using Slutsky's equation and the results for the uncompensated wage elasticities we have

$$(\frac{\partial H(s)}{\partial W(s)})_S\cdot\frac{W(s)}{H(s)} = \frac{\partial H(s)}{\partial W(s)}\cdot\frac{W(s)}{H(s)} - H(s)\cdot\frac{\partial H(s)}{\partial A(0)}\cdot\frac{W(s)}{H(s)}$$

$$= \frac{\partial H(s)}{\partial W(s)}\cdot\frac{W(s)}{H(s)} - H(s)\cdot W(s)\cdot\frac{\partial \ln H(s)}{\partial A(0)} = \delta+\gamma(s) - H(s)\cdot W(s)\cdot\kappa,$$

and

$$(\frac{\partial H(t)}{\partial W(s\neq t)})_S\cdot\frac{W(s\neq t)}{H(t)} = \frac{\partial H(t)}{\partial W(s\neq t)}\cdot\frac{W(s\neq t)}{H(t)} - H(s\neq t)\cdot\frac{\partial H(t)}{\partial A(0)}\cdot\frac{W(s\neq t)}{H(t)}$$

$$= \frac{\partial H(t)}{\partial W(s\neq t)}\cdot\frac{W(s\neq t)}{H(t)} - H(s\neq t)\cdot W(s\neq t)\cdot\frac{\partial \ln H(t)}{\partial A(0)} = \gamma(s) - H(s\neq t)\cdot W(s\neq t)\cdot\kappa,$$

for the own- and cross-compensated wage elasticity, respectively.

2.2 Some Additional Intuitions regarding the Model

Some additional clarifying thoughts on the model are worth mentioning:

- The underlying concept of dynamic equilibrium is key to understand the model we are dealing with: this is the set of equilibrium time profiles for labor supply and consumption determined for a given value of initial wealth or net worth, A_0, and for given values of wages, $W(t)$, and prices, $P(t)$, for all periods t.

- It is natural to think that simple efficiency considerations would dictate that the individual will work the most during periods when the wage (the opportunity cost of leisure) is highest. To use a quote from Weiss (1972, Economic Journal):

  "Work is allocated according to lifetime wage differentials. There is a positive association between changes in wages and changes in hours of work. The dynamic effect of a wage increase is clearly less ambiguous than its static effect. The reason for this difference is the separation of consumption and production (work) decisions due to the existence of savings. Efficiency requires the transfer of effort to period with high earning capacity."

- But there are more things going on: First, compound interest will probably encourage hard work first, bank his earnings, and reduce labor supply with advancing age, with leisure as a normal good. Second, "future effort seems less painful when viewed from the present." With a positive subjective rate of time preference people prefer to enjoy leisure now rather than later. This will tend to see labor supply rising as time passes.

- There are three forces working at the same time: the *efficiency* effect, the *interest rate* effect, and the *time preference effect*.

- The first force is proportional to the size of the rate of change in the wage. The other two force can be summarized by the difference between ρ and r. The rate of change of labor supply over time due to these effects will have the sign of this difference. Assuming leisure is a normal good.

- Clearly if $\rho = r$ then hours of work and wages will move in the same direction over the life cycle. But they can move in opposite directions for at least part of the life cycle if that is not the case.

Some simple Comparative Dynamics might be in order:

- It is worth considering the effect of an unanticipated increase in initial net worth. Provided that consumption and leisure are normal good at all dates, this will increase consumption and leisure at all dates.

- If wages in a given period increase, but this increase was anticipated, the profile of work and consumption is not affected. But if this increase is unanticipated then we will observe a wealth effect and a substitution effect. The former is for sure going to reduce labor supply, the second will increase labor supply in the surprise period, and depending on the cross-substitution effect will increase or reduce leisure at times different from that t.

- Notice the importance of seeing that anticipated effects are just movements along a given labor supply profile, and should be analyzed as an issue of dynamic equilibrium, not comparative dynamics.

A clarification regarding the particular assumptions of the model usually solved:

- Intertemporal additive separability seems fairly innocuous, but it has some specific implications about behavior. If we assume as we will do that leisure time at any given date is a normal good, this assumption will mean leisure times at different dates are net substitutes. This means that cross-substitution effects of wage rate changes on hours of work are negative. This is the case if we keep utility constant through a reduction in initial assets.

2.3 Stochastic Wage Profile

Now we consider a variant of the original problem with uncertain wage profile. We will focus on the implications for expression (13) and the possibilities to estimate the intertemporal elasticity of substitution, δ.

The individual's problem is characterized by the following Langrangian given initial wealth, $A(0)$:

$$L = E\sum_{t=0}^{K}(\frac{1}{1+\rho})^t\{U(C(t),H(t))+\lambda_t\cdot((1+r_t)\cdot A_{t-1}+W(t)\cdot H(t)-P(t)\cdot C(t)-A_t)\} \tag{18}$$

where

$(1+r_t)\cdot A_{t-1}+W(t)\cdot H(t)-P(t)\cdot C(t)=A_t$

is the intertemporal asset/savings constraint. Assuming that preferences are strictly concave in consumption and strictly convex in hours worked, the optimal interior path of consumption and hours worked satisfies the following necessary and sufficient conditions

$$\frac{\partial L}{\partial C(t)}=0 \Leftrightarrow \frac{\partial U}{\partial C(t)}=\lambda_t\cdot P(t),\ \ t=0,\ldots,K, \tag{19}$$

$$\frac{\partial L}{\partial H(t)}=0 \Leftrightarrow \frac{\partial U}{\partial H(t)}=-\lambda_t\cdot W(t),\ \ t=0,\ldots,K, \tag{20}$$

$$\frac{\partial L}{\partial A_t}=0 \Leftrightarrow -\lambda_t\cdot(\frac{1}{(1+\rho)^t})+E(t)\{\lambda_{t+1}\frac{1+r_{t+1}}{(1+\rho)^{t+1}}=0\},\ \ t=0,\ldots,K, \tag{21}$$

$$\Leftrightarrow \lambda_t = E_t\{\lambda_{t+1}\frac{1+r_{t+1}}{1+\rho}\},$$

where E_t is the expectation operator conditional on the information set at time t.

The last condition requires that the discounted expected marginal utility of wealth is constant over the life cycle. Using the wealth constraint, it is easy to see that the marginal utility of current wealth is a function of current and future (expected) wages, current and future prices, current and future interest rates, and preferences:

$$\lambda(t) = \lambda(W(t), W(t+1), ..., W(K); P(t), ..., P(K); A_t; r_t, ..., r_K; \rho; X(t), ..., X(K); \varepsilon^*(t), ..., \varepsilon^*(K)).$$

We assume that expectations are formed rationally

$$\lambda_{t+1} = E_t \lambda_t + \mu_{t+1}, \tag{22}$$

where $E_t \lambda_{t+1}$ is the expected marginal utility of wealth in $t+1$ where the expectation is formed using all relevant information available at the beginning of period t. Hence, μ_{t+1} is an innovation (meaning expectation error orthogonal to the information at t) to the marginal utility of wealth that arrived between t and $t+1$.

Assuming rational expectations and that interest rates are deterministic, expression (21) can be rearranged (after applying natural logarithm and approximation) to take the following form:

$$\frac{1+r_{t+1}}{1+\rho} \frac{\lambda_{t+1}}{\lambda_t} = 1 + \frac{1+r_{t+1}}{1+\rho} \frac{\mu_{t+1}}{\lambda_t} \tag{23}$$

$$\Rightarrow \ln \lambda_{t+1} - \ln \lambda_t \approx \rho - r_{t+1} + \frac{1+r_{t+1}}{1+\rho} \frac{\mu_{t+1}}{\lambda_t}.$$

What are the empirical implications of allowing individuals to be uncertain about future wages?

To answer this question we adopt MaCurdy's specification. Combining expressions (13) and (14) and taking first differences yields,

$$\Delta \ln H_i(t) = \alpha' \cdot \Delta \ln \lambda_i(t) + b \cdot \Delta t + \delta \cdot \Delta \ln W_i(t) + \Delta \varepsilon_i(t) \tag{24}$$

$$= \alpha' \cdot (b + \rho - r_t) + \delta \cdot \Delta \ln W_i(t) + \alpha' \cdot \frac{1 + r_t}{1 + \rho} \frac{\mu_t}{\lambda_{t-1}} + \Delta \varepsilon_i(t),$$

where $\alpha' = \frac{1}{\alpha - 1}$.

How does the presence of $\mu_i(t)$ affect the strategy to estimate the intertemporal elasticity of substitution, δ?

Estimates of δ will be biased if wage changes between $t-1$ and t are unanticipated by the individual since $\mu_i(t)$ and $\Delta \ln W_i(t)$ will be correlated. The bias is downwards, i.e. an estimate of δ from a regression based on equation (24) will underestimate the elasticity of substitution (why? you should think about this).

To obtain an unbiased estimate of δ we need variables that are correlated with $\Delta \ln W_i(t)$ but not with $\mu_i(t)$. In other words we need a measure of the anticipated wage change between $t-1$ and t. A proposed instrument that can be used to construct the suggested Instrumental Variables Estimator is the lagged wage change, $\Delta \ln W_i(t-1)$.

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