A Strategic Analysis of the War Against Transnational Terrorism*

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Abstract. We study a two stage game in which a transnational terrorist organization interacts with an arbitrary number of countries that may differ in their political or economic power, their military effectiveness, the benefit from cooperating against terrorism and the value they assign to damage. Only a subset of countries that emerges endogenously takes proactive measures to fight the terrorist, while all countries incur defensive expenditures to protect their soil. We characterize analytically the pure strategy subgame perfect equilibrium of the game and show how the equilibrium strategies depend on the key model parameters. We provide an algorithm to find the endogenous set of cooperating countries based on their benefit from cooperation and their political/economic power.

Keywords: Transnational Terrorism, Proactive and Defensive measures, Strategic Interactions, Conflict Resolution.

JEL Classification: D74

1. Introduction

The study of terrorism has been an active field of research since the early 1970s, with a strong increase in interest after the events of 9/11. Recently, a growing number of researchers have applied game theory to study transnational terrorism.[1] However, important questions have been left unaddressed.[2] For example, a crucial normative issue is the determination of the level of counterterrorist measures that countries should implement, depending on their specific characteristics, such as the benefit they obtain from cooperating against terrorism, their military effectiveness or their political/economic power. In addition, examples of empirical facts that have been left unexplained, like why only a coalition of countries and not all of them proactively fights terrorism. In the present paper, we build a model to analyze these issues. In particular, we characterize which coalition of countries emerges endogenously to fight terrorism.

Our framework captures the strategic interaction of transnational terrorist organizations, such as Al-Qaeda, with an arbitrary number of countries that are potential targets for terrorist attacks. It differs from the literature in several important aspects. First, it assumes that both the terrorist organization and the countries act strategically. Whereas the terrorist organization chooses the optimal allocation of resources to attack the countries, countries can take proactive and defensive measures against the terrorist organization. Tracking the terrorist assets or invading and destroying their bases are some examples of proactive measures

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[1] Terrorism is transnational when incidents in a given country involve perpetrators, victims, institutions, governments, or citizens of another country.

[2] An extensive review of the literature can be found in Enders and Sandler (2006). See also Enders and Sandler (1995), Sandler and Arce (2003) and Sandler and Enders (2004, 2005).

used by countries to aggressively fight terrorism. Strengthening border and transportation security, preparing for biological and chemical attacks and protecting energy supply lines are examples of defensive measures, which are used to reduce the vulnerability of potential home-front targets against terrorist attacks. Second, countries can have different characteristics that influence their optimal decisions and determine endogenously which ones proactively fight the terrorist and which are a better target for a terrorist attack. In particular, countries can differ in their political/economic power, their military effectiveness, the benefit that each one obtains from cooperating against terrorism and their valuation of casualties and damage that can be inflicted on them upon a successful terrorist attack. It is assumed that all countries are potential targets for a terrorist attack, whether or not they proactively fight the terror organization.

The game has two stages. In the first stage, a predetermined subset of countries choose independently and simultaneously their proactive effort levels. As a result, the initial resources of the terrorist are downgraded to a level that depends on their combined efforts. In the second stage, the terrorist allocates its remaining resources to attack the countries while, at the same time, the countries choose their defensive measures. It is important to note that we do not study a zero sum game. For any country, the payoff is assumed to decrease with the magnitude of the damage caused by a successful terrorist attack and the costs that it incurs when exerting counterterrorism measures. Further, it increases in the benefit that this country obtains from cooperating against terrorism. As for the payoff of the terrorist organization, the definition of its ultimate objective is a complicated and debated issue. In this paper, we focus on its intermediate goal, which we believe is to increase the number of its supporters, as they constitute most of the resources of the organization.[3] In particular, we assume that the organization seeks to achieve its goal by inflicting the highest possible level of damage through successful attacks, since this directly translates into publicity and thereby attracts more followers.[4] This aspect is consistent with the empirical evidence that successful attacks stimulate the recruitment of terrorists.

We start by analyzing the case in which the benefits from cooperation against terrorism are all non-negative. In this case, we show that there exists a unique pure strategy subgame perfect equilibrium of the static game. Further, the equilibrium strategies are characterized as a function of the key model parameters. Some of the interesting findings are as follows. First, a country that assigns a relatively high value to casualties and damage will exert higher proactive efforts and sufficiently high defensive measures so that the terrorist allocates a smaller proportion of his resources to attack this country. Second, a country is better off having a higher benefit from cooperating against terrorism and all countries are better off even if the benefit from cooperating is higher for just one country (but not lower for the others). In this case, this latter country exerts a higher proactive effort that reduces the expected damage that the terrorist can inflict. Third, a country is better off if it has a lower political/economic power, since it is a less attractive target for the terrorist and can free ride on other countries by exerting lower counterterrorist efforts. We also find that a country that is militarily ineffective is in general worse off, since it is subject to higher expected damage. In this case, the country also free rides on other countries by substituting some of the proactive effort with defensive actions.These findings are consistent with empirical evidence, that smaller countries tend to rely on efforts of more powerful and militarily effective countries when the terrorist threat is common to all. Surprisingly, we

[3] This is consistent with the empirical evidence that common types of terrorist attacks, such as the ones implemented through skyjackings or bombings, do not use costly physical resources, as documented by Enders and Sandler (2006) and the Economist (2003).

[4] Faria and Arce (2005) argue that each terrorist success makes it easier for groups such as Al-Qaeda to recruit new members. The impact of media coverage on the economic damage of terrorist attacks is studied in Melnick and Eldor (2006).

also find situations under which the payoff of a country may increase when it becomes less effective militarily, provided that the initial terrorist resources are sufficiently high and the country is not too powerful in political/economic terms. Under the latter two conditions, the saving in military costs offsets the increase in defense costs. Next, we compute the cooperative outcome in which the countries that take proactive measures act jointly so as to maximize their total payoffs, while all countries continue to choose their defensive efforts independently. It is shown that each country in the cooperating group increases its proactive effort, while all countries decrease their defensive effort. This implies that in the cooperative solution countries who do not proactively fight terror free ride more on other countries than in the non cooperative solution.

One of the most important contributions of the paper is the endogenous formation of the coalition of countries that proactively fight the terrorist. Every country with a positive benefit from cooperation will belong to this coalition. However, we show that some countries with a negative benefit from cooperation (for example countries where the majority of the population oppose a war against terrorism) may still proactively fight the terrorist, since they constitute an attractive target for the organization. A simple algorithm to find the coalition of countries that proactively fight the terrorist is provided. This set is determined by the ratios of the benefit countries obtain from cooperation to their political/economic power. The countries are ranked according to these ratios and a cutoff level (which may be negative) is endogenously determined. Only countries with a ratio above this level will be proactive. We show that even countries with a negative benefit from cooperation may belong to this coalition if their political/economic power is large.

As mentioned above, our work relates to the literature on terrorism and game theory, which has studied issues such as government concessions (mostly regarding hostages), the terrorists' choice of a target and the governments' counterterrorist responses.[5] In particular, we relate more closely to the literature on counterterrorism which has mostly studied two country 2 by 2 games in which the terrorist is not a strategic player (see e.g. Lee (1988) and Arce and Sandler (2005)). Only recently several authors have analyzed extensive form games. For example, Sandler and Siqueira (2006) study a model in which two targets choose independently defensive measures (version 1) or proactive measures (version 2) against the non-strategic terrorist organization. While the level of these measures determines the probability of successful attacks, they are not chosen optimally. Bandyopadhyay and Sandler (2009) (B-S hereafter) study a two stage game of counterterrorism where both proactive and defensive measures are taken by two countries with a common terrorist threat. There are significant differences between their paper and the present one. Our model deals with an arbitrary number of countries, while B-S consider only two. We make the terrorist and the targeted countries active agents, while B-S assume that the terrorist is not strategic. In contrast to our model, B-S allow countries to have foreign assets. This curbs the desire of the home country to shift attacks abroad through enhanced defense. Moreover, it increases the desire of countries for proactive measures, since a country may be at risk at home and abroad. Finally, we endogenize the subset of countries that proactively fight terrorism based on their benefit from cooperation and political/economic power.

Rosendorff and Sandler (2004) study a static two player game in which the government chooses first the proactive effort level and the terrorist chooses afterwards the type of attack (normal or a spectacular). Trajtenberg (2006) studies a model with three sectors: a non strategic terrorist, targets in a given country that choose defensive measures and a govern-

[5] Some of the earlier papers include Atkinson, Sandler and Tschirhart (1987), Lapan and Sandler (1988), Sandler, Tschirhart and Cauley (1983), Sandler and Lapan (1988) and Selten (1988). Recently, Bueno de Mesquita (2005a, 2005b) uses a formal model to study the strategic interaction between terrorists and the government, with the focus on government concessions. In addition Lowenberg and Mathews (2008) the government's decision on how to allocate protective measures across different targets within a single country.

ment who chooses the proactive effort level. The battle between a government and a terrorist has been studied by Jacobson and Kaplan (2007). The authors study a model where the terrorist decides how often to attack and the government decides how often to execute targeted killings as part of its counterterrorism campaign. Whereas the previous authors only study two player games in which the actions of the terrorist are discrete, the government can only choose proactive counterterrorist measures. Finally, Hausken (2008) studies the interaction between a terrorist and a government. On the one hand, the terrorist only defends his assets but does not attack the government. On the other hand, the government tries to destroy the terrorist's asset, which grows from period to period.

Our paper differs from the previous literature in several important aspects. It emphasizes the strategic interaction between the terrorist and an arbitrary number of targets. Moreover, apart from considering differences in the cost of counterterrorist measures and in the value that countries assign to terrorist attacks (also studied in some of the previous literature), we also consider differences in the benefit from cooperating against terrorism and in the political and economic power of countries. These aspects play a crucial role in the study of the endogenous formation of the coalition of countries that take proactive measures against the terrorist organization.

Several extensions would be worthwhile exploring. First, one could extend the model to a dynamic setting to study under what conditions the terrorist can be defeated or will coexist with the countries. Second, one could incorporate asymmetric information regarding the resources of the terrorist or the probability of successful attacks by the terrorist organization.[6] We leave this for further research.

2. The Model

The model consists of n countries and a terrorist organization. The set of countries is denoted by $N = \{1, 2, ..., n\}$ and the terrorist organization is denoted by T. The latter has some initial resources that are denoted by R_0. Every country in N takes defensive measures to protect its soil against terrorist attacks but only a subset $N_0 \subseteq N$ of countries takes proactive measures against T. For simplicity of the exposition and the proofs, the set N_0 is fixed at this stage but it will be endogenously determined later on. The countries in N and T are engaged in a two stage game $G(N, N_0, R_0)$ that is described below.

At the first stage of the game $G(N, N_0, R_0)$, each country $i \in N_0$ chooses a proactive military effort level x_i to fight T inside or outside the country, where $x_i = 0$ if $i \notin N_0$. The effort levels $(x_i)_{i \in N_0}$ are chosen simultaneously and independently. As a result, the total resources of T are reduced from R_0 to R, where

$$R \equiv R\left(R_0, \sum_{i \in N_0} x_i\right) < R_0.$$

At the second stage of the game $G(N, N_0, R_0)$, R becomes commonly known. Each country $i \in N$ chooses a defensive effort level y_i, which represents the monetary investment to protect the country against a terror attack. Simultaneously, T allocates the total resources R among the N countries, that is, it chooses $(R_i)_{i \in N}$ such that[7]:

$$R = \sum_{i=1}^{n} R_i, \text{ with } R_i \geq 0. \tag{1}$$

[6] Some examples of papers introducing asymmetric information into a model of terrorism are Arce and Sandler (2007), Arce and Sandler (2010), Lapan and Sandler (1993) and Overgaard (1994).

[7] In contrast, Powell (2007) considers a setting, closely related to Blotto games, in which a defender (a government) allocates scarce resources in several of its own sites and the terrorist decides which target to attack.

Note that this implies that T may attack any country in N and not only the ones in the cooperating group N_0. Upon a successful attack, we assume that the damage that T can inflict on a country $i \in N$ is random with mean

$$\lambda_i \equiv \lambda\left(P_i, R_i, y_i\right), \tag{2}$$

where P_i measures the political and/or economic power of $i \in N$. We also assume that countries assign different monetary valuations to a unit of damage inflicted by T, and we denote these valuations by $(v_i)_{i \in N}$.

The countries in N_0 obtain a political/economic benefit from their cooperation against T. For example, these countries might obtain international recognition, trade benefits or other economic advantages from other countries in the group. We assume that the benefit from cooperation for country $i \in N_0$ depends on the contribution level x_i to the total proactive effort and we denote it by $b_i(x_i)$. Clearly, if $N_0 = \{i\}$, country i is the only one fighting T and there is no cooperation. Therefore, we assume that $b_i(x_i) \equiv 0$ if $N_0 = \{i\}$. The benefit from cooperation comes at the expense of a monetary cost. In particular, every country $i \in N_0$ pays a cost of $c_i(x_i)$ for providing a proactive effort level x_i.

The expected payoff of country $i \in N$ is equal to:

$$\pi_i\left(N_0\right) = \left\{ \begin{array}{cc} b_i(x_i) - c_i(x_i) - y_i - v_i \lambda\left(P_i, R_i, y_i\right) & \text{if } i \in N_0 \\ -y_i - v_i \lambda\left(P_i, R_i, y_i\right) & \text{if } i \notin N_0 \end{array} \right\} \tag{3}$$

The expected (and actual) payoff function of a country depends on several components, which are all expressed in monetary terms. First, if a country belongs to the cooperating group N_0, she will exert a proactive effort level of x_i at a cost of $c_i(x_i)$, obtaining a benefit of $b_i(x_i)$. These two terms are not present if i does not belong to the cooperating group. Second, the payoffs depend on the defensive expenditure y_i. Finally, the expected payoff decreases linearly with $v_i \lambda_i$, where λ_i represents the expected damage and v_i is the monetary valuation that country i assigns to each unit of damage. Since the damage inflicted by the terrorist is assumed to be a random variable, the actual payoff decreases linearly with the actual damage times the damage valuation v_i.

Regarding the terrorist, we assume that its expected payoff is the total expected damage it inflicts:

$$\pi_T = \sum_{i=1}^{n} \lambda\left(P_i, R_i, y_i\right) = \sum_{i=1}^{n} \lambda_i, \tag{4}$$

where $R_i \geq 0$, $\sum_{i=1}^{n} R_i = R$ and $R \equiv R\left(R_0, \sum_{i \in N_0} x_i\right)$.

The definition of the ultimate objective function for the terrorist organization is a complicated and debated issue. In this paper, we focus on its intermediate goal. For the fundamentalist terrorism that we have observed in the last decade, we believe that this goal is to increase the number of supporters, which constitute most of the resources of the organization. Further, this is achieved by inflicting the highest possible level of damage through successful attacks, which will expose the organization, through free media channels all over the world, and attract more followers. For example, the attacks of 9/11 attracted a huge attention from the media and were broadcast live all over the world, a fact that could be exploited by the terrorists to gain more support and recruit more members.

One might argue that the relevant objective function of maximizing the number of followers should be:

$$\pi_T = R\left(R_0, \sum_{i \in N_0} x_i\right) + \rho \sum_{i=1}^{n} \lambda_i \tag{5}$$

where $\rho > 0$. The payoff function has two components. The term $\rho \sum_{i=1}^{n} \lambda_i$ is the expected number of new followers, which is proportional to the total damage that T inflicts on the countries in N. The term $R\left(R_0, \sum_{i\in N_0} x_i\right)$ is the number of current followers who have survived the first stage attack on T by the countries in N_0. Note, however, that $R\left(R_0, \sum_{i\in N_0} x_i\right)$ is taken as given in the second stage of the game and T takes no action in the first stage of the game, in which x_i is determined. Hence, the equilibrium of the game does not change whether the objective function of T is (5) or simply (4). In other words, the two approaches of maximizing the expected damage or the expected number of followers are equivalent.

3. Characterization of the Equilibrium

This section studies the pure strategy subgame perfect equilibrium (SPE) of the game $G(N, N_0, R_0)$. To be able to characterize the equilibrium explicitly, we assume specific functional forms. Moreover, with a general functional form, the existence of an equilibrium is not guaranteed, even if the payoff functions are concave in the strategic variables. Note that this is due to the fact that the second stage equilibrium strategies $(y_i, R_i)_{i=1}^n$ may depend on the first stage decision variables $(x_i)_{i=1}^n$ in a way in which the first stage payoffs as functions of $(x_i)_{i=1}^n$ may neither be concave nor continuous.[8]

The functional forms we assume are the following. First, after the cooperating countries in N_0 have attacked T, the total resources available to T are:

$$R\left(R_0, \sum_{i\in N_0} x_i\right) = R_0 \exp\left(-\epsilon \sum_{i\in N_0} x_i\right),\ \epsilon \geq 0 \tag{6}$$

The previous equation reflects that the resources of the terrorist are decreasing in the total proactive effort, with the parameter ϵ representing the total military effectiveness of the countries in N_0. Note that $R_0 - R$, which is the reduction in the resources of T due to the attack by countries in N_0, is concave in x_i $\left(\text{and in} \sum_{i\in N_0} x_i\right)$.

The expected damage inflicted by T on country $i \in N$ is assumed to be increasing in both the political/economic power of $i \in N$ and the resources allocated by T, while it decreases in the defensive expenditure of $i \in N$:

$$\lambda_i = \lambda(P_i, R_i, y_i) = \frac{P_i R_i}{y_i} \tag{7}$$

Note that $-\lambda_i$ is increasing and concave in y_i.

The cost of exerting a proactive effort level of x_i is assumed to be quadratic in x_i:

$$c_i(x_i) = \frac{1}{2}\gamma_i x_i^2, \tag{8}$$

where γ_i represents the military effectiveness of country $i \in N_0$. In particular, country i is more effective the smaller is γ_i. Last, the benefit from cooperation is assumed to be linear in x_i:

$$b_i(x_i) = b_i x_i, \text{ where } b_i = 0 \text{ if } N_0 = \{i\}. \tag{9}$$

It is important to note that b_i can take both positive and negative values. Whereas a positive value of b_i implies that the direct benefit of a country from fighting the terrorist is positive, a negative value reflects a cost for being part of the cooperating group. This might be due to internal political costs of actively fighting T. Nevertheless, a country with a negative b_i might still find it beneficial to join the cooperating group if by doing so it can

[8] If $(x_1, ..., x_n)$ lies in a compact space and one can guarantee that there exists a unique equilibrium of the second stage of the game $\left(R_i^*(x_1, ..., x_n), y_i^*(x_1, ..., x_n)\right)_{i=1}^n$ for every $(x_1, ..., x_n)$, then R_i^* and y_i^* are continuous in $(x_1, ..., x_n)$ and the existence of a mixed strategy equilibrium of $G(N, N_0, R_0)$ is guaranteed.

increase their net equilibrium payoff. For instance, a country with a larger political/economic power P_i is an attractive target for T and it has a strong incentive for degrade its resources. Later on, we will extend the analysis to allow for $b_i < 0$. In what follows, we study the case in which $b_i \geq 0$ for all $i \in N$.

With the functional forms defined above, the expected payoff functions become:

$$\pi_T = \sum_{i=1}^{n} \frac{P_i R_i}{y_i} \tag{10}$$

$$\pi_i(N_0) = \left\{ \begin{array}{ll} b_i x_i - \frac{1}{2}\gamma_i x_i^2 - y_i - \frac{v_i P_i R_i}{y_i} & \text{if } i \in N_0 \\ -y_i - \frac{v_i P_i R_i}{y_i} & \text{if } i \notin N_0 \end{array} \right\} \tag{11}$$

The existence of a unique SPE in pure strategies for the game $G(N, N_0, R_0)$ is established in Proposition 1.

Proposition 1. *The game $G(N, N_0, R_0)$ has a unique pure strategy SPE, which is characterized by:*

1. $x_i^* = \frac{b_i}{\gamma_i} + \frac{\frac{P_i}{\gamma_i}\left[x^* - \sum_{k\in N_0}\frac{b_k}{\gamma_k}\right]}{\sum_{k\in N_0}\frac{P_k}{\gamma_k}}$ *for all $i \in N_0$, where x^* is the unique solution of*

2. $x^* = \sum_{k\in N_0} x_k^* = \sum_{k\in N_0}\frac{b_k}{\gamma_k} + \frac{\epsilon\left(\sum_{k\in N_0}\frac{P_k}{\gamma_k}\right)R_0^{0.5}\exp\left(-\frac{\epsilon}{2}x^*\right)}{\left(\sum_{k=1}^{n}\frac{P_k}{v_k}\right)^{0.5}}$.

3. $y_i^* = \frac{P_i R_0^{0.5}\exp\left(-\frac{\epsilon}{2}x^*\right)}{\left(\sum_{k=1}^{n}\frac{P_k}{v_k}\right)^{0.5}}$, *$i \in N$. Therefore, for all $i, j \in N$, $\frac{y_i^*}{y_j^*} = \frac{P_i}{P_j}$.*

4. $R_i^* = \frac{P_i R_0 \exp(-\epsilon x^*)}{v_i\left(\sum_{k=1}^{n}\frac{P_k}{v_k}\right)}$, *$i \in N$. Therefore, for all $i, j \in N$, $\frac{R_i^*}{R_j^*} = \frac{\frac{P_i}{v_i}}{\frac{P_j}{v_j}}$.*

Note that, in equilibrium, the ratio of $\frac{P_i}{y_i}$ does not depend on i and the targets are equally attractive to T. Also, T allocates his available resources to every country i in proportion to this ratio. That is, the proportion of resources that T allocates to country i is increasing in its political/economic power P_i and is decreasing in the value v_i that that country assigns to a unit of damage. The proof of all propositions throughout the paper are relegated to the Appendix. Propositions 2-6 characterize the properties of the SPE equilibrium of $G(N, N_0, R_0)$. To illustrate them, we also provide numerical examples with $N = \{1, 2\}$.

Proposition 2. *An increase in the total initial resources R_0 of T: (1) Increases the defensive efforts of all countries in N and the proactive efforts of all countries in N_0. (2) Increases the expected damage inflicted on all countries and decreases their payoffs. (3) Increases the payoff of T.*

Table 1 provides a numerical example that illustrates the effects of increasing the initial resources of the terrorist R_0 from 1 to 5.

Table 1: A change in R_0

R_0	R	R_1	R_2	y_1	y_2	x_1	x_2	x	π_1	π_2	π_T
1	0.963	0.321	0.642	0.566	1.133	0.156	0.213	0.37	-1.13	-2.26	1.70
5	4.720	1.573	3.146	1.254	2.508	0.225	0.350	0.57	-2.51	-5.04	3.76

$N_0 = N$, $P_1 = 1$, $P_2 = 2$, $v_i = 1$, $\gamma_i = 1$, $b_i = 0.1$, $\epsilon = 0.1$.

The table shows that an increase in R_0 leads to an increase in the defensive effort level of all countries to protect their soil against terrorist attacks. Furthermore, the countries also increase their proactive effort levels to fight a stronger terrorist organization. In spite of the higher counterterrorist measures, the first order effect dominates and T allocates almost 5 times more resources to each country. As a result, the expected damage increases for every country, leading to a decrease in their payoffs and to an increase in the payoff of T.

Proposition 3. *An increase in v_i for $i \in N$: (1) Increases the defensive effort of i and its proactive effort* (if $i \in N_0$). *As a result, there is a shift of terrorist resources from i to any other country $j \in N$, who in turn increases its defensive effort and its proactive effort* (if $j \in N_0$). *(2) Decreases the expected damage on i and increases it for any other country $j \in N$. Nevertheless, the payoff decreases for all countries. (3) Increases the total proactive effort x and decreases the available resources of T as well as his payoff.*

An important implication of Proposition 3 is that an increase in the damage valuation of any country makes all countries and the terrorist worse off, demonstrating that the game is not a zero sum game. Proposition 3 also implies that a higher damage valuation of a given country increases its defensive expenditures and diverts terrorist resources to other countries.[9] This is consistent with the empirical evidence that higher defensive efforts by a given country divert attacks to softer targets, as documented by Enders and Sandler (1993,2004,2006b).

Proposition 4. *An increase in P_i for $i \in N$: (1) Increases the proactive effort of i* (if $i \in N_0$) *and its defensive effort. (2) Increases the percentage of resources allocated by T to i and decreases it for every other country. As a result, there is an increase in the expected damage for i and a decrease for every other country. Consequently, the payoff decreases for i and increases for every other country. (3) Decreases both the defensive effort of every other country $j \in N$ and its proactive effort* (if $j \in N_0$).

The implications of Proposition 4 are consistent with the empirical evidence that more powerful countries exert bigger proactive efforts, whereas smaller countries free ride on them. Table 3 illustrates the effects of an increase in the political/economic power of the second country from 2 to 5.

Table 2: A change in P_2

P_1	P_2	R	R_1	R_2	y_1	y_2	x_1	x_2	x	π_1	π_2	π_T
1	2	0.963	0.321	0.642	0.566	1.133	0.156	0.213	0.37	-1.13	-2.26	1.70
1	5	0.957	0.159	0.797	0.399	1.996	0.139	0.299	0.43	-0.79	-4.00	2.39

$N_0 = N$, $v_i = 1$, $\gamma_i = 1$, $b_i = 0.1$, $\epsilon = 0.1$, $R_0 = 1$

If country 2 is more powerful, it exerts bigger proactive and defensive efforts. As a consequence, the terrorist allocates more resources to it, increasing the expected damage and decreasing the payoff for this country. In contrast, since less resources are allocated to the first country, this country decreases both the defensive and proactive effort levels. In other words, when the second country is more powerful, the first country free rides. As a result, the expected damage decreases for the first country, leading to an increase in its payoff, while the payoff decreases for country two.[10]

Proposition 5. *An increase in γ_i for $i \in N_0$: (1) Decreases the proactive effort of i, which is substituted with a higher defensive effort. To compensate, every other country $j \in N_0$ increases the proactive effort, namely i free rides on j. (2) Decreases the total proactive effort x and increases the available resources of T, who allocates more to each country. This induces an increase in the defensive effort of every country. (3) Increases the expected damage of all countries. The payoff of every country decreases and the payoff of*

[9] The first two parts of this proposition are anticipated by Sandler and Lapan (1988) and Sandler and Siqueira (2006).

[10] Note that this could be offset by other factors such as costs margins or terorrist biases to attack certain countries (see Bandyopadhyay and Sandler (2009)). Here, we would also like to point out that power is not the only factor inducing terrorist attacks. Other factors such as grievances are also important. As an example, Israel isn't very powerful compared to US, France or Britain but it attracts a lot of terrorism.

T increases. (4) If $\frac{P_i}{\gamma_i} < \frac{1}{2}\sum_{k\in N_0}\frac{P_k}{\gamma_k}$ and R_0 is sufficiently large, the payoff of i increases even though he becomes less effective. Otherwise, the payoff of i decreases.

An interesting implication of the proposition is that there are situations under which the payoff of the country that becomes less effective increases. To shed light on this phenomenon, assume that $\frac{P_i}{\gamma_i} < \frac{1}{2}\sum_{k\in N_0}\frac{P_k}{\gamma_k}$. Note that we can interpret $\frac{P_i}{\gamma_i}$ as the power of country $i \in N$, which takes into account both the political/economic power and its military effectiveness. If the previous condition is satisfied, the power of a country i is less than one half of the total power of the countries in N_0. In other words, country i is not too powerful, in the sense that it does not have the majority of the total power.

Consider now a per unit increase in γ_i for $i \in N_0$. On the one hand, the per unit increase in γ_i has a direct effect that increases the proactive effort cost $\frac{1}{2}\gamma_i x_i^2$ by $\frac{1}{2}x_i^2$. On the other hand, since $\frac{\partial x_i}{\partial \gamma_i} < 0$, the country reduces its proactive effort level and the proactive cost decreases by $\gamma_i x_i \left|\frac{\partial x_i}{\partial \gamma_i}\right|$. It turns out that, when x_i is sufficiently large, this indirect effect is also quadratic in x_i with a coefficient that is larger than $\frac{1}{2}$ (in absolute value) if $\frac{P_i}{\gamma_i} < \frac{1}{2}\sum_{k\in N_0}\frac{P_k}{\gamma_k}$. In other words, the latter condition is needed to obtain a cost saving effect for a sufficiently large x_i as a result of the increase in γ_i. Note that a decrease in x_i has a negative effect on the benefit from cooperation $b_i x_i$, but this effect is only linear in x_i. Thus, a sufficiently large x_i implies an improvement in the payoff of i. Finally, note that x_i increases indefinitely with the initial resources R_0 of T. Hence, if $\frac{P_i}{\gamma_i} < \frac{1}{2}\sum_{k\in N_0}\frac{P_k}{\gamma_k}$, a sufficiently large R_0 leads to an increase in the payoff of i as a result of an increase in γ_i. On the other hand, if either R_0 is small or if $\frac{P_i}{\gamma_i} > \frac{1}{2}\sum_{k\in N_0}\frac{P_k}{\gamma_k}$, then the payoff of country i decreases unambiguously if it becomes less effective.

Proposition 6. *An increase in the cooperation benefit b_i for $i \in N_0$: (1) Increases the proactive effort of i and decreases the proactive effort of every other country in N_0. (2) Increases the total proactive effort and decreases the total available resources of T. As a result, all countries decrease their defensive efforts. (3) Decreases the expected damage and increases the payoff of every country, while the payoff of T declines.*

Proposition 6 has the interesting implication that a higher benefit from cooperation for a given country benefits all other countries, since it will make the terrorist weaker.

4. The cooperative outcome

In this section, we compare the optimal strategies of the countries in N_0 under binding cooperation with the non-cooperative strategies derived in Proposition 1. The outcome under binding cooperation is obtained when the countries in N_0 maximize the sum of their payoffs:

$$\Pi = \sum_{i\in N_0}\left[b_i x_i - \frac{1}{2}\gamma_i x_i^2 - y_i - \frac{v_i P_i R_i}{y_i}\right] \tag{12}$$

First, Π is maximized over $(y_i)_{i\in N_0}$ for a given $(x_i)_{i\in N_0}$, taking into account the simultaneous defensive efforts of all other countries in $N\backslash N_0$ and the allocation of resources R_0 of T. Then, Π is maximized over $(x_i)_{i\in N_0}$. The following proposition characterizes the cooperative allocations.

Proposition 7. *Suppose that $|N_0| \geq 2$. Let $(x_i^c)_{i\in N_0}$ and $(y_i^c)_{i\in N_0}$ be the cooperative solution and let $(x_i^*)_{i\in N_0}$ and $(y_i^*)_{i\in N_0}$ be the non-cooperative solution. Then, $y_i^c < y_i^*$ for all $i \in N$ and $x_i^c > x_i^*$ for all $i \in N_0$.*

The proposition asserts that under cooperation every country in N_0 exerts bigger proactive efforts and every country in N exerts lower defensive efforts. This implies that every country in $N\backslash N_0$ free rides more on the countries in N_0 in the cooperative solution than in the non-cooperative solution.[11]

[11] The suboptimality of the non-cooperative outcome has been established in different settings (see e.g.

5. Negative Benefits and Sustainable Cooperating Groups

In the previous section, we have assumed that $b_i \geq 0$ for all $i \in N$. Under this assumption, Proposition 1 asserts that the equilibrium level x_i^* is positive for all $i \in N_0$ for any arbitrary set N_0 of N, including $N_0 = N$. In this section, we (i) allow for $b_i < 0$ for some or all $i \in N$; (ii) obtain the set N_0 endogenously and (iii) provide an algorithm to compute an equilibrium set N_0 of cooperating countries.

Note that a country i with $b_i < 0$ may have an incentive to proactively fight T if, for example, P_i is sufficiently large. However, if P_i is relatively small and the country is not an attractive target for the terrorist, then it will be optimal for this country not to fight T. Namely, the equilibrium first order condition for this country will result in x_i^* being negative. Potentially, one could allow for negative proactive efforts, with the interpretation that a country i with $x_i^* < 0$ supports rather than fights T, in the sense that it provides T with a military support of the magnitude $|x_i^*|$. However, this is not consistent with our assumption that T may target any country in N. Given this, we restrict our attention to the case in which the proactive efforts are constrained to be non-negative. In other words, a country can either actively fight T or not, but no country in N supports T. Under this assumption, the first order conditions are not satisfied as equalities for some countries and the analysis is more complicated.

To analyze this latter case, consider the same game that we have described in the previous section but let $N_0 = N$ and denote this game by $G(N, R_0)$. The only difference is that $G(N, R_0)$ allows for $b_i < 0$ for some or all $i \in N$. In the first stage, countries choose simultaneously their proactive efforts $(x_1, ..., x_n)$, with $x_i \geq 0$ for all $i \in N$. T is then attacked with the military power $\sum_{i \in N} x_i$ and its resources are reduced to:

$$R = R_0 \exp\left(-\epsilon \sum_{i \in N} x_i\right).$$

In the second stage, $(x_1, ..., x_n)$ becomes commonly known and the countries in N choose simultaneously their defensive expenditures $(y_1, ..., y_n)$, while T chooses the allocation of resources to attack the countries.

Consider now a subgame perfect equilibrium of $G(N, R_0)$, assuming that it exists. Let $N_0 = \{i \in N | x_i^* > 0\}$, where $(x_1^*, ..., x_n^*)$ are the first stage equilibrium actions of the countries in N. Clearly, N_0 is the set of countries which in equilibrium proactively fight T. We refer to N_0 as a *sustainable* set. It is important to note that finding a sustainable set is not an obvious task. To see this, consider the case in which, when solving the first order equilibrium conditions (as was done in Proposition 1), some country i with $b_i < 0$ may be best off choosing $x_i < 0$. Let $\widetilde{x}_i$ for $i \in N$ be the level of proactive effort that solves the first order conditions, namely

$$\widetilde{x}_i = \frac{b_i}{\gamma_i} + \frac{\frac{P_i}{\gamma_i}\left[\widetilde{x} - \sum_{k \in N} \frac{b_k}{\gamma_k}\right]}{\sum_{k \in N} \frac{b_k}{\gamma_k}}$$

where $\widetilde{x}$ is the unique solution x of

$$x = \sum_{k \in N} \frac{b_k}{\gamma_k} + \frac{\epsilon\left(\sum_{k \in N} \frac{P_k}{\gamma_k}\right) R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x\right)}{\left(\sum_{k=1}^{n} \frac{P_k}{v_k}\right)^{0.5}}$$

Further, let $\widetilde{N}_0 = \{i \in N | \widetilde{x}_i > 0\}$. Then $\widetilde{N}_0$ may not be sustainable. The reason is that a country $j \in \widetilde{N}_0$ with $b_j < 0$ may choose $\widetilde{x}_j > 0$ just because the total effort $\widetilde{x}$ is small.

Sandler and Lapan (1988) and Sandler and Siqueira (2006)).

However, when the countries $i \in N \backslash \widetilde{N}_0$ shift from $\widetilde{x}_i < 0$ to $\widetilde{x}_i = 0$ to satisfy their feasibility constraints, then $j \in \widetilde{N}_0$ may be better off reducing its level of proactive effort to $x_j = 0$, hence leaving $\widetilde{N}_0$. Our next goal is to provide an algorithm that finds a sustainable set. The algorithm is described in what follows.

Algorithm to Find a Sustainable Set. Let N_0 be an arbitrary subset of N and let

$$\widetilde{x}_i(N_0) = \frac{b_i}{\gamma_i} + \frac{\frac{P_i}{\gamma_i}\left[\widetilde{x}(N_0) - \sum_{k \in N_0} \frac{b_k}{\gamma_k}\right]}{\sum_{k \in N_0} \frac{b_k}{\gamma_k}}$$

where $\widetilde{x}(N_0)$ is the unique solution x of

$$x = \sum_{k \in N_0} \frac{b_k}{\gamma_k} + \frac{\epsilon\left(\sum_{k \in N_0} \frac{P_k}{\gamma_k}\right) R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x\right)}{\left(\sum_{k=1}^{n} \frac{P_k}{v_k}\right)^{0.5}}$$

Under the assumption that only countries in N_0 exert positive efforts, $\widetilde{x}_i(N_0)$ is the unconstrained equilibrium proactive effort of $i \in N_0$ (see Proposition 1). In particular, if $\widetilde{x}_i(N_0) > 0$ for all $i \in N_0$, then $\widetilde{x}_i(N_0) = x_i^*(N_0)$. However, $\widetilde{x}_i(N_0)$ may be negative if $b_i < 0$, in which case it will be different from the equilibrium effort level $x_i^*(N_0) = 0$.

Suppose without loss of generality that $\frac{b_1}{P_1} \geq \frac{b_2}{P_2} \geq ... \geq \frac{b_n}{P_n}$. Further, for every k, $1 \leq k \leq n$, let $N_0^k = \{1, ..., k\}$. It is shown in the Appendix that:

$$\widetilde{x}_k\left(N_0^k\right) \leq 0 \text{ implies that } \widetilde{x}_m\left(N_0^k + m\right) \leq 0 \text{ for all } m > k.$$

To construct a sustainable set, we note first that $\widetilde{x}_1\left(N_0^1\right) > 0$, since $b_1 = 0$ if $N_0 = \{1\}$. Next, we start with N_0^1 and we add one by one the countries 2,3,...etc. until the first time we find a k such that $\widetilde{x}_{k+1}\left(N_0^{k+1}\right) \leq 0$. If we do not find such a k, then $N_0 = N$ is the sustainable set. Otherwise, N_0^k is sustainable. Proposition 8 below states the existence of a pure strategy subgame perfect equilibrium for the game $G(N, R_0)$.

Proposition 8. *(1) The game $G(N, R_0)$ has a pure strategy subgame perfect equilibrium and a sustainable set can be derived using the algorithm above. (2) In every subgame perfect equilibrium, the sustainable set of cooperating countries is non empty. (3) Any sustainable set must include every country $i \in N$ with $b_i \geq 0$. In particular, if $b_i \geq 0$ for all $i \in N$, then $N_0 = N$ is the only sustainable set.*

To illustrate how the sustainable set of countries depends on some of the model parameters, we provide some numerical examples with five countries, namely $N = \{1, 2, 3, 4, 5\}$. Table 3 displays the level of proactive effort of the different countries for different values of the initial resources of the terrorist, with the last row displaying the corresponding set N_0. As explained earlier, only the countries with a positive level of proactive effort belong to the cooperating group. All examples assumes that only the first country has a positive benefit of cooperating against T.

When $R_0 = 1$, implying that the initial resources of the terrorist are relatively small, only the country with a positive benefit from cooperation belongs to the cooperating group. However, as R_0 increases, more countries find it beneficial to fight T. In particular, $N_0 = \{1, 2\}$ if $R_0 = 10$, $N_0 = \{1, 2, 3\}$ if $R_0 = 100$ and $N_0 = \{1, 2, 3, 4, 5\}$ if $R_0 = 900$. This example shows that some countries with a negative benefit from cooperation might join the cooperating group if the terrorist is sufficiently powerful.

Table 3: Sustainable cooperating set for a different R_0

Proactive Effort with $N = 5$ and different R_0

Country	b_i	$R_0 = 1$	$R_0 = 10$	$R_0 = 100$	$R_0 = 900$
1	0.1	0.14	0.23	0.52	1.20
2	-0.1	0	0.03	0.32	1.00
3	-0.2	0	0	0.22	0.90
4	-0.5	0	0	0	0.60
5	-1	0	0	0	0.10
N_0		$\{1\}$	$\{1,2\}$	$\{1,2,3\}$	$\{1,2,3,4,5\}$

$\epsilon = 0.1$, $v_i = 1$, $\gamma_i = 1$ and $P_i = 1$ for all $i \in N$

Table 4 displays again the level of proactive efforts of the different countries in N for different levels of ϵ, which represents how effective countries are in reducing the resources of the terrorist. As before, the last row displays the corresponding set N_0. The table shows that more countries find it beneficial to join the cooperating group as ϵ increases. In particular, $N_0 = \{1,2\}$ if $\epsilon = 0.05$, $N_0 = \{1,2,3\}$ if $\epsilon = 0.1$, $N_0 = \{1,2,3,4\}$ if $\epsilon = 0.11$ and $N_0 = \{1,2,3,4,5\}$ if $\epsilon = 0.2$. Again, countries with a negative benefit from cooperation might join the cooperating group if their proactive effort can reduce the resources of the terrorist more effectively.

Table 4: Sustainable cooperating set for a different ϵ

Proactive Effort with $N = 5$ and different ϵ

Country	b_i	$\epsilon = 0.05$	$\epsilon = 0.1$	$\epsilon = 0.11$	$\epsilon = 0.2$
1	0.1	0.14	0.19	0.20	0.28
2	-0.01	0.03	0.08	0.09	0.17
3	-0.05	0	0.04	0.05	0.13
4	-0.1	0	0	0.007	0.08
5	-0.15	0	0	0	0.03
N_0		$\{1,2\}$	$\{1,2,3\}$	$\{1,2,3,4\}$	$\{1,2,3,4,5\}$

$R_0 = 5$, $v_i = 1$, $\gamma_i = 1$ and $P_i = 1$ for all $i \in N$

Table 5 illustrates the effects on the sustainable set when there is a change in the political/economic power of countries. In this example, the power increases for the three countries with the lowest benefit from cooperation. When the power of each country is equal to 1, only the country with a positive benefit fights T. However, when the power of the last three countries increases considerably, they find it beneficial to join N_0, since they are more likely to be a target.

Table 5: Sustainable cooperating set for a different P_i

External Effort with $N = 5$ and different P_i

Country	b_i	P_i	x_i	P_i	x_i
1	0.1	1	0.14	1	0.10
2	-0.1	1	0	1	0
3	-0.2	1	0	100	0.19
4	-0.5	1	0	200	0.28
5	-1	1	0	300	0.17
N_0			$\{1\}$		$\{1,3,4,5\}$

$R_0 = 1$, $\epsilon = 0.1$, $v_i = 1$, $\gamma_i = 1$

6. Conclusions

This paper studies a sequential game in which an arbitrary set of countries interacts with a transnational terrorist organization. Whereas the terrorist organization distributes its available resources to attack the different countries, countries choose both the optimal level of defensive measures to protect their soil against terrorist attacks and the level of proactive measures to aggressively fight terrorism. Every country exerts defensive measures but the set of countries that proactively fights the terrorist emerges endogenously in equilibrium. Clearly, defensive measures taken by a country may divert attacks to other countries. In addition, proactive measures taken by a country that reduce the terrorist threat for other countries create positive externalities but lead to free rider problems. These issues constitute an important part of our analysis.

We first characterize analytically the subgame perfect equilibrium of the game. In particular, we study how the strategic actions depend on the key model parameters, namely, the political/economic power of countries, their military effectiveness, the value they assign to a per unit of damage resulting from a terrorist attack and the benefit they obtain from cooperating against the terrorist. We also provide sufficient conditions for a defeat of the terrorist or a given country as well as an algorithm to obtain the endogenous cooperating group that proactively fights the terrorist. This algorithm can be used whether or not the benefits from cooperation are negative.

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APPENDIX

Throughout the Appendix, we use the following notation:

$$C_i = \frac{P_i}{v_i},\ B_i = \frac{P_i}{\gamma_i},\ B_{-i} = \sum_{k\in N_0\backslash i} B_k,\ A_i = \frac{b_i}{\gamma_i}$$

with $C = \sum_{i=1}^{n} C_i$, $B = \sum_{i\in N_0} B_i$, $B_{-i} = \sum_{k\in N_0\backslash i} B_k$, $A = \sum_{i\in N_0} A_i$ and $A_{-i} = \sum_{k\in N_0\backslash i} A_k$.

Proof of Proposition 1. Recall that:

$$\pi_T = \sum_{i=1}^{n} \frac{P_i R_i}{y_i} \tag{13}$$

and, for every $i \in N$,

$$\pi_i = \left\{ \begin{array}{cc} b_i x_i - \frac{1}{2}\gamma_i x_i^2 - y_i - \frac{v_i P_i R_i}{y_i}, & i \in N_0 \\ -y_i - \frac{v_i P_i R_i}{y_i}, & i \in N\backslash N_0 \end{array} \right\} \tag{14}$$

At the second stage of the game, each country $i \in N$ maximizes π_i over y_i given $(x_i)_{i\in N_0}$ and $(R_i)_{i\in N}$. Since π_i is strictly concave in y_i for every R_i and $(x_i)_{i\in N_0}$, it is maximized over y_i if:

$$\frac{\partial \pi_i}{\partial y_i} = -1 + \frac{P_i R_i v_i}{y_i^2} = 0.$$

Thus,

$$y_i = (P_i R_i v_i)^{0.5}. \tag{15}$$

Simultaneously with the choices $(y_i)_{i\in N}$, T maximizes π_T over $(R_i)_{i\in N}$ given $(x_i)_{i\in N_0}$ and subject to $\sum_{i=1}^{n} R_i = R$ and $R_i \geq 0$ for all $i \in N$. We first maximize π_T without the last non-negativity constraint. Define the Lagrangian function:

$$L = \pi_T - \mu\left(\sum_{i=1}^{n} R_i - R\right).$$

The first order conditions imply that $\frac{\partial L}{\partial R_i} = \frac{P_i}{y_i} - \mu = 0$ for all $i \in N$,and $\sum_{i=1}^{n} R_i = R$. Thus, $\mu = \frac{P_i}{y_i}$ and, by (15) $R_i = \frac{1}{\mu^2}\frac{P_i}{v_i} = \frac{C_i}{\mu^2}$.This also implies that $R = \frac{C}{\mu^2}$. Therefore,

$$R_i^* = \frac{C_i}{C} R = \frac{P_i R_0 \exp(-\epsilon x^*)}{v_i\left(\sum_{k=1}^{n} \frac{P_k}{v_k}\right)}. \tag{16}$$

Since $R_i^* > 0$, the constraint $R_i \geq 0$ is not binding and equation (16) proves part 4 of Proposition 1. Substituting (16) into (15), we have:

$$y_i^* = P_i\left(\frac{R}{C}\right)^{0.5}. \tag{17}$$

Note that $\frac{P_i}{y_i^*} = \left(\frac{C}{R}\right)^{0.5}$ and does not depend on i. This implies that T is indifferent as to how to allocate his resources R. Irrespective of the allocation, he will obtain a total expected damage of $\sum_{i=1}^{n} R_i \left(\frac{C}{R}\right)^{0.5}$, that is

$$\pi_T^* = (CR)^{0.5}. \tag{18}$$

or, equivalently,

$$\pi_T^* = \left(\sum_{k=1}^{n} \frac{P_k}{v_k}\right)^{0.5} R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x^*\right). \tag{19}$$

At the first stage of the game, countries in N_0 choose $(x_i)_{i\in N_0}$ simultaneously and independently. Let

$$s_i = 2P_i \left(\frac{R_0}{C}\right)^{0.5}. \tag{20}$$

By (14), (16), (17) and (20), we have for $i \in N_0$:

$$\pi_i = b_i x_i - \frac{1}{2}\gamma_i x_i^2 - s_i \exp\left(-\frac{\epsilon}{2}\sum_{k\in N_0} x_k\right). \tag{21}$$

Hence, $\frac{\partial \pi_i}{\partial x_i} = b_i - \gamma_i x_i + \frac{\epsilon}{2} s_i \exp\left(-\frac{\epsilon}{2}\sum_{k\in N_0} x_k\right)$. Since $\frac{\partial^2 \pi_i}{\partial x_i^2} < 0$ and $\frac{\partial \pi_i}{\partial x_i}(x_i = 0) > 0$, the equilibrium x_i is the solution of $\frac{\partial \pi_i}{\partial x_i} = 0$. That is,

$$x_i = \frac{b_i}{\gamma_i} + \frac{\epsilon}{2}\frac{s_i}{\gamma_i} \exp\left(-\frac{\epsilon}{2}\sum_{k\in N_0} x_k\right). \tag{22}$$

Let $x = \sum_{i\in N_0} x_k$. By (22) $x = \sum_{i\in N_0} \frac{b_i}{\gamma_i} + \frac{\epsilon}{2}\sum_{i\in N_0} \frac{s_i}{\gamma_i} \exp\left(-\frac{\epsilon}{2}x\right) \equiv f(x)$. We first show that the equation $f(x) = x$ has a unique solution. Let $g(x) = f(x) - x$. It is easy to see that $g(0) = f(0) > 0$, $g'(x) < 0$ for all x and $g(x) < 0$ for x sufficiently large. Since g is continuous in x, there exists a unique solution x^* such that $g(x^*) = 0$ or $f(x^*) = x^*$, namely,

$$x^* = \sum_{i\in N_0} \frac{b_i}{\gamma_i} + \frac{\epsilon}{2}\sum_{i\in N_0} \frac{s_i}{\gamma_i} \exp\left(-\frac{\epsilon}{2}x^*\right) = \sum_{i\in N_0} \frac{b_i}{\gamma_i} + \epsilon\left(\sum_{i\in N_0} \frac{P_i}{\gamma_i}\right) \frac{R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x^*\right)}{C^{0.5}} \tag{23}$$

as claimed. By (22) and (23),

$$x_i^* = \frac{b_i}{\gamma_i} + \frac{\epsilon}{2}\frac{s_i}{\gamma_i} \exp\left(-\frac{\epsilon}{2}x^*\right). \tag{24}$$

By (23) and (24)

$$\frac{\left[x^* - \sum_{i\in N_0} \frac{b_k}{\gamma_k}\right]}{\sum_{i\in N_0} \frac{s_i}{\gamma_i}} = \left[x_i^* - \frac{b_i}{\gamma_i}\right]\frac{\gamma_i}{s_i},$$

implying that:

$$x_i^* = \frac{b_i}{\gamma_i} + \frac{\frac{P_i}{\gamma_i}\left[x^* - \sum_{i\in N_0} \frac{b_k}{\gamma_k}\right]}{\sum_{i\in N_0} \frac{P_k}{\gamma_k}}. \tag{25}$$

The previous equation, together with (20), proves parts 1 and 2 of Proposition 1. Finally, by (13), (14), (16), (17), (20) and (24), it can be verified that

$$y_i^* = \frac{P_i R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x^*\right)}{\left(\sum_{k=1}^{n} \frac{P_k}{v_k}\right)^{0.5}}, i \in N$$

The previous equation proves part 3 of Proposition 1. In addition, it can be shown that

$$y_i^* = \frac{\gamma_i x_i^* - b_i}{\epsilon},\ i \in N_0 \text{ and } \epsilon > 0 \tag{26}$$

$$R_i^* = \frac{(y_i^*)^2}{v_i P_i},\ i \in N \tag{27}$$

$$\pi_i = \left\{ \begin{array}{ll} \left[b_i - \frac{2\gamma_i}{\epsilon}\right] x_i^* - \frac{1}{2}\gamma_i (x_i^*)^2 + \frac{2b_i}{\epsilon}, & i \in N_0 \\ -2y_i^* = -\frac{2P_i R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x^*\right)}{\left(\sum_{k=1}^n \frac{P_k}{v_k}\right)^{0.5}}, & i \in N \backslash N_0 \end{array} \right\} \tag{28}$$

$$\lambda_i^* = \frac{P_i}{v_i} \frac{R_0^{0.5}}{\left(\sum_{k=1}^n \frac{P_k}{v_k}\right)^{0.5}} \exp\left(-\frac{\epsilon}{2}x^*\right),\ i \in N. \tag{29}$$

The previous equation implies that $\frac{\lambda_i^*}{\lambda_j^*} = \frac{\frac{P_i}{v_i}}{\frac{P_j}{v_j}}$ for all $i, j \in N$.$\blacksquare$

The proofs of Propositions 2-6 follow directly from Lemmas 1-4 below. Throughout the proofs, whenever we refer to the results of Proposition 1, we include equations (26-29).

Lemma 1.

$$\begin{aligned}
(1)\ \frac{\partial x^*}{\partial P_i} &= \left\{ \begin{array}{ll} \frac{\epsilon R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x^*\right)\left[2\frac{v_i}{\gamma_i} - \frac{B}{C}\right]}{v_i\left[\epsilon^2 B R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x^*\right) + 2C^{0.5}\right]}, & i \in N_0 \\ -\frac{\epsilon R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x^*\right) B}{v_i C\left[\epsilon^2 R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x^*\right) B + 2C^{0.5}\right]} < 0, & i \notin N_0 \end{array} \right\} \\
\frac{\partial x^*}{\partial P_i} &< 0 \text{ if } i \notin N_0. \text{ For } i \in N_0,\ \frac{\partial x^*}{\partial P_i} < 0 \text{ iff } \frac{v_i}{\gamma_i} < \frac{B}{2C}. \\
(2)\ \frac{\partial x^*}{\partial v_i} &= \frac{\epsilon R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x^*\right) P_i B}{\left[\epsilon^2 R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x^*\right) B + 2C^{0.5}\right] C v_i^2},\ i \in N. \\
(3)\ \frac{\partial x^*}{\partial \gamma_i} &= -\frac{2\left[b_i C^{0.5} + \epsilon P_i R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x^*\right)\right]}{\left[\epsilon^2 R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x^*\right) B + 2C^{0.5}\right] \gamma_i^2},\ i \in N_0. \\
(4)\ \frac{\partial x^*}{\partial b_i} &= \frac{2C^{0.5}}{\left[\epsilon^2 R_0^{0.5} \exp\left(-\frac{\epsilon}{2}x^*\right) B + 2C^{0.5}\right] \gamma_i},\ i \in N_0. \\
(5)\ \frac{\partial x^*}{\partial R_0} &= \frac{\epsilon B \exp\left(-\frac{\epsilon}{2}x^*\right)}{2C^{0.5} R_0^{0.5} + \epsilon^2 R_0 \exp\left(-\frac{\epsilon}{2}x^*\right) B}.
\end{aligned}$$

The proof follows directly from Proposition 1.

Lemma 2. (1) $\frac{\partial x^*}{\partial R_0} > 0$; For $i \in N$, $\frac{\partial x^*}{\partial v_i} > 0$; For $i \in N_0$, $\frac{\partial x^*}{\partial b_i} > 0; \frac{\partial x^*}{\partial \gamma_i} < 0; \frac{\partial x^*}{\partial P_i} < 0$ iff $i \notin N_0$ or $i \in N_0$ and $\frac{v_i}{\gamma_i} < \frac{B}{2C}$. (2) For all $i, j \in N_0$, $k \in N$, $j, k \neq i$, $\frac{\partial x_i^*}{\partial v_i} > 0; \frac{\partial x_i^*}{\partial v_k} > 0; \frac{\partial x_i^*}{\partial P_i} > 0; \frac{\partial x_i^*}{\partial P_k} < 0; \frac{\partial x_i^*}{\partial b_i} > 0; \frac{\partial x_i^*}{\partial b_j} < 0; \frac{\partial x_i^*}{\partial \gamma_i} < 0; \frac{\partial x_i^*}{\partial \gamma_j} > 0; \frac{\partial x_i^*}{\partial R_0} > 0$. (3) For all $i, j \in N_0$, $k \in N$, $j, k \neq i$, $\frac{\partial y_i^*}{\partial v_i} > 0; \frac{\partial y_i^*}{\partial v_k} > 0$; $\frac{\partial y_i^*}{\partial P_i} > 0$; $\frac{\partial y_i^*}{\partial P_k} < 0$; $\frac{\partial y_i^*}{\partial b_i} < 0$; $\frac{\partial y_i^*}{\partial b_j} < 0$; $\frac{\partial y_i^*}{\partial \gamma_i} > 0; \frac{\partial y_i^*}{\partial \gamma_j} > 0; \frac{\partial y_i^*}{\partial R_0} > 0$. (4) For all $i, j \in N$, $j \neq i$, $\frac{\partial y_i^*}{\partial R_0} > 0$; $\frac{\partial \lambda_i^*}{\partial v_i} < 0; \frac{\partial \lambda_i^*}{\partial v_j} > 0; \frac{\partial \lambda_i^*}{\partial P_i} > 0; \frac{\partial \lambda_i^*}{\partial P_j} < 0; \frac{\partial \lambda_i^*}{\partial R_0} > 0$. (5) For all $i \in N_0$, $\frac{\partial \lambda_i^*}{\partial b_i} < 0$; $\frac{\partial \lambda_i^*}{\partial \gamma_i} > 0$. (6) For all $i \in N$ and $j \in N_0$, $i \neq j$, $\frac{\partial \lambda_i^*}{\partial b_j} < 0; \frac{\partial \lambda_i^*}{\partial \gamma_j} > 0$.

Lemma 3. (1) For all $i, j \in N$, $j \neq i$, $\frac{\partial R_i^*}{\partial v_i} < 0$ and $\frac{\partial R_i^*}{\partial v_j} > 0$. (2) For all $i \notin N_0$, $\frac{\partial R_i^*}{\partial P_i} > 0$ and for all $i \in N_0$, $\frac{\partial R_i^*}{\partial P_i} > 0$ if $B_i < \frac{1}{2}B$. (3) For all $i \in N_0$, $\frac{\partial R_i^*}{\partial b_i} < 0$ and for all $i \in N$ and $j \in N_0$, $\frac{\partial R_i^*}{\partial b_j} < 0$. (4) For all $i \in N_0$, $\frac{\partial R_i^*}{\partial \gamma_i} > 0$ and for all $i \in N$ and $j \in N_0$, $\frac{\partial R_i^*}{\partial \gamma_j} > 0$.

The proof of the previous two Lemmas is tedious but straightforward and it is therefore not included but can be provided by the authors upon request.

Lemma 4. In what follows, we let $\pi_i^* = \pi_i^*(N_0)$. (1) For all $i, j \in N$ and $j \neq i$, $\frac{\partial \pi_i^*}{\partial v_i} < 0$ and $\frac{\partial \pi_i^*}{\partial v_j} < 0$. (2) For all $i, j \in N$ and $j \neq i$, $\frac{\partial \pi_i^*}{\partial P_i} < 0$ and $\frac{\partial \pi_i^*}{\partial P_j} > 0$. (3) For all $i \in N_0$ $\frac{\partial \pi_i^*}{\partial b_i} > 0$; For all $i \in N$, $j \in N_0$, $i \neq j$, $\frac{\partial \pi_i^*}{\partial b_j} > 0$. (4) The payoff of $i \in N_0$ depends on γ_i as follows: $\frac{\partial \pi_i^*}{\partial \gamma_i} < 0$ if $B_i \geq \frac{1}{2}B$. If $B_i < \frac{1}{2}B$, $\exists\ \widehat{R}_0$ s.t. $\frac{\partial \pi_i^*}{\partial \gamma_i} < 0$ if $R_0 < \widehat{R}_0$ and $\frac{\partial \pi_i^*}{\partial \gamma_i} > 0$ if $R_0 > \widehat{R}_0$. For every $i \in N$, $j \in N_0$, $j \neq i$, $\frac{\partial \pi_i^*}{\partial \gamma_j} < 0$.

Proof of Lemma 4. We first prove that $\frac{\partial \pi_i^*}{\partial v_i} < 0$ and $\frac{\partial \pi_i^*}{\partial v_j} > 0$ for all $i, j \in N$, $i \neq j$. By Proposition 1, if $i \in N_0$

$$\frac{\partial \pi_i^*}{\partial v_i} = \left(b_i - \frac{2\gamma_i}{\epsilon} - \gamma_i x_i^*\right) \frac{\partial x_i^*}{\partial v_i}.$$

Since $\frac{\partial x_i^*}{\partial v_i} > 0$, it is sufficient to show that $\left(b_i - \frac{2\gamma_i}{\epsilon} - \gamma_i x_i^*\right) < 0$. By Proposition 1,

$$b_i - \frac{2\gamma_i}{\epsilon} - \gamma_i x_i^* = -\frac{2\gamma_i}{\epsilon} - \gamma_i \frac{B_i}{B}\left(x^* - A\right) < 0.$$

implying that $\frac{\partial \pi_i^*}{\partial v_i} < 0$. Suppose that $i \in N \backslash N_0$. By Proposition 1, it is sufficient to prove that $\frac{\partial\left[\frac{\exp(-\epsilon x^*)}{C}\right]}{\partial v_i} > 0$. This is equivalent to $\frac{P_i}{v_i^2} > \epsilon C \frac{\partial x^*}{\partial v_i}$. By Lemma 1, this is equivalent to

$$\frac{\epsilon^2 B R_0^{0.5} \exp\left(-\frac{\epsilon}{2} x^*\right)}{\epsilon^2 B R_0^{0.5} \exp\left(-\frac{\epsilon}{2} x^*\right) + 2C^{0.5}} < 1$$

which is clearly true. By Proposition 1, for $i \in N_0$, $\frac{\partial \pi_i^*}{\partial v_j} = -\gamma_i \left(\frac{2}{\epsilon} + \frac{B_i}{B}\left(x^* - A\right)\right) \frac{\partial x_i^*}{\partial v_j}$.Since $\frac{\partial x_i^*}{\partial v_j} > 0$, it follows that $\frac{\partial \pi_i^*}{\partial v_j} < 0$. For $i \in N \backslash N_0$, the proof that $\frac{\partial \pi_i^*}{\partial v_j} < 0$ is exactly the same as the proof that $\frac{\partial \pi_i^*}{\partial v_i} < 0$.

We prove next that for all $i, j \in N$, $j \neq i$, $\frac{\partial \pi_i^*}{\partial P_i} < 0$ and $\frac{\partial \pi_i^*}{\partial P_j} > 0$. Suppose first that $i \in N_0$. By Proposition 1,

$$\frac{\partial \pi_i^*}{\partial P_i} = \left(b_i - \frac{2\gamma_i}{\epsilon} - \gamma_i x_i^*\right) \frac{\partial x_i^*}{\partial P_i} = \left(-\frac{2\gamma_i}{\epsilon} - \gamma_i \frac{B_i}{B}\left(x^* - A\right)\right) \frac{\partial x_i^*}{\partial P_i}.$$

Since $\frac{\partial x_i^*}{\partial P_i} > 0$, it follows that $\frac{\partial \pi_i^*}{\partial P_i} < 0$. By Proposition 1, $\frac{\partial \pi_i^*}{\partial P_j} = -\gamma_i \left(\frac{2}{\epsilon} + \frac{B_i}{B}\left(x^* - A\right)\right) \frac{\partial x_i^*}{\partial P_j}$. Since $\frac{\partial x_i^*}{\partial P_j} < 0$, it follows that $\frac{\partial \pi_i^*}{\partial P_j} > 0$. Suppose next that $i \in N \backslash N_0$. By Proposition 1, $\frac{\partial \pi_i^*}{\partial P_i} < 0$ iff $\frac{\partial\left[\frac{P_i^2 \exp(-\frac{\epsilon}{2} x^*)}{C}\right]}{\partial P_i} > 0$. Equivalently, $\epsilon \frac{\partial x^*}{\partial P_i} < \frac{2}{P_i} - \frac{1}{v_i C}$. By Lemma 1, $\frac{\partial x^*}{\partial P_i} < 0$ and $\frac{2}{P_i} - \frac{1}{v_i C} = \frac{2C - C_i}{P_i C} > 0$. Hence, the proof is complete. Suppose next that $i \in N \backslash N_0$ and $j \in N_0$. Then, $\frac{\partial \pi_i^*}{\partial P_j} > 0$ iff $\frac{\partial\left[\frac{\exp(-\frac{\epsilon}{2} x^*)}{C^{0.5}}\right]}{\partial P_j} < 0$. Equivalently, $\frac{\partial\left[\frac{\exp(-\epsilon x^*)}{C}\right]}{\partial P_j} < 0$ or $\epsilon \frac{\partial x^*}{\partial P_j} > -\frac{1}{v_j C}$.By Lemma 1 applied to $j \in N_0$, this is equivalent to

$$\frac{\epsilon^2 B R_0^{0.5} \exp\left(-\frac{\epsilon}{2} x^*\right)\left[2\frac{v_j}{\gamma_j} - \frac{B}{C}\right]}{v_j\left[\epsilon^2 B R_0^{0.5} \exp\left(-\frac{\epsilon}{2} x^*\right) + 2C^{0.5}\right]} > -\frac{1}{v_j C}.$$

The last inequality is equivalent to $2\frac{v_j}{\gamma_j} \epsilon^2 R_0^{0.5} \exp\left(-\frac{\epsilon}{2} x^*\right) C > -2C^{0.5}$which clearly holds.

We prove next that $\frac{\partial \pi_i^*}{\partial b_i} > 0$ for $i \in N_0$ and $\frac{\partial \pi_i^*}{\partial b_j} > 0$ for $i \in N$, $j \in N_0$, $i \neq j$. By

Proposition 1,

$$\frac{\partial \pi_i^*}{\partial b_i} = \left(\frac{b_i}{\gamma_i} - \frac{2}{\epsilon} - x_i^*\right)\gamma_i \frac{\partial x_i^*}{\partial b_i} + x_i^* + \frac{2}{\epsilon}.$$

Hence, $\frac{\partial \pi_i^*}{\partial b_i} > 0$ iff $\gamma_i \frac{\partial x_i^*}{\partial b_i} < 1 + \frac{\frac{b_i}{\gamma_i}}{\left(-\frac{b_i}{\gamma_i}+\frac{2}{\epsilon}+x_i^*\right)}$.By (??), $\frac{\partial x_i^*}{\partial b_i} = \frac{B_{-i}}{\gamma_i B} + \frac{P_i}{\gamma_i B}\frac{\partial x^*}{\partial b_i} = \frac{B_{-i}}{\gamma_i B} + \frac{B_i}{B}\frac{\partial x^*}{\partial b_i}$.Consequently, $\frac{\partial \pi_i^*}{\partial b_i} > 0$ iff $\frac{B_{-i}}{B} + \frac{\gamma_i B_i}{B}\frac{\partial x^*}{\partial b_i} < 1 + \frac{\frac{b_i}{\gamma_i}}{\left(-\frac{b_i}{\gamma_i}+\frac{2}{\epsilon}+x_i^*\right)}$ or $\frac{\partial x^*}{\partial b_i} < \frac{1}{\gamma_i} + \frac{Bb_i}{B_i\gamma_i^2\left(-\frac{b_i}{\gamma_i}+\frac{2}{\epsilon}+x_i^*\right)}$.By Lemma 1, $\frac{\partial x^*}{\partial b_i} < \frac{1}{\gamma_i}$ and hence $\frac{\partial \pi_i^*}{\partial b_i} > 0$. By Proposition 1, $\frac{\partial \pi_i^*}{\partial b_j} = -\gamma_i\left(\frac{2}{\epsilon} + \frac{B_i}{B}(x^* - A)\right)\frac{\partial x_i^*}{\partial b_j}$. Since $\frac{\partial x_i^*}{\partial b_j} < 0$, it follows that $\frac{\partial \pi_i^*}{\partial b_j} > 0$.

We prove next that for $i \in N_0$, $\frac{\partial \pi_i^*}{\partial \gamma_i} < 0$ unless $B_i < \frac{1}{2}B$ and R_0 is sufficiently large. Also $\frac{\partial \pi_i^*}{\partial \gamma_j} < 0$ for $i \in N$, $j \in N_0$, $j \neq i$ always holds. By Proposition 1,

$$\begin{aligned}\frac{\partial x_i^*}{\partial \gamma_i} &= -\frac{b_i}{\gamma_i^2} - \frac{B_i B_{-i}[x^* - A]}{\gamma_i B^2} + \frac{B_i\left[\frac{\partial x^*}{\partial \gamma_i} + \frac{b_i}{\gamma_i^2}\right]}{B} \\ &= \frac{B_i}{B}\frac{\partial x^*}{\partial \gamma_i} - \frac{B_{-i}x_i^*}{\gamma_i B}.\end{aligned} \tag{30}$$

By Proposition 1,

$$\frac{\partial x^*}{\partial \gamma_i} = -\frac{b_i}{\gamma_i^2} - \frac{\epsilon B_i R_0^{0.5}\exp\left(-\frac{\epsilon}{2}x^*\right)}{\gamma_i C^{0.5}} - \frac{\epsilon^2 B R_0^{0.5}\exp\left(-\frac{\epsilon}{2}x^*\right)}{2C^{0.5}}\frac{\partial x^*}{\partial \gamma_i}.$$

Let $M = \frac{\epsilon R_0^{0.5}\exp\left(-\frac{\epsilon}{2}x^*\right)}{C^{0.5}}$. Then $\frac{\partial x^*}{\partial \gamma_i} = -\frac{b_i}{\gamma_i^2} - \frac{MB_i}{\gamma_i} - \frac{\epsilon BM}{2}\frac{\partial x^*}{\partial \gamma_i}$.Since $M = \frac{x^* - A}{B}$, then

$$\begin{aligned}\frac{\partial x^*}{\partial \gamma_i} &= -\frac{b_i + MB_i\gamma_i}{\gamma_i^2\left(1 + \frac{\epsilon BM}{2}\right)} = -\frac{b_i + \frac{B_i\gamma_i}{B}[x^* - A]}{\gamma_i^2\left(1 + \frac{\epsilon}{2}[x^* - A]\right)} \\ &= -\frac{x_i^*}{\gamma_i\left[1 + \frac{\epsilon B}{2B_i}\left(x_i^* - \frac{b_i}{\gamma_i}\right)\right]}.\end{aligned} \tag{31}$$

The two inequalities (30) and (31) imply:

$$\begin{aligned}\frac{\partial x_i^*}{\partial \gamma_i} &= -\frac{B_{-i}x_i^*}{\gamma_i B} - \frac{B_i x_i^*}{\gamma_i B\left[1 + \frac{\epsilon B}{2B_i}\left(x_i^* - \frac{b_i}{\gamma_i}\right)\right]} \\ &= -\frac{x_i^*\left[1 + \frac{\epsilon B_{-i}}{2B_i}\left(x_i^* - \frac{b_i}{\gamma_i}\right)\right]}{\gamma_i\left[1 + \frac{\epsilon B}{2B_i}\left(x_i^* - \frac{b_i}{\gamma_i}\right)\right]}.\end{aligned} \tag{32}$$

By Proposition 1, $\frac{\partial \pi_i^*}{\partial \gamma_i} = -\gamma_i\left(x_i^* - \frac{b_i}{\gamma_i} + \frac{2}{\epsilon}\right)\frac{\partial x_i^*}{\partial \gamma_i} - \frac{2x_i^*}{\epsilon} - \frac{1}{2}(x_i^*)^2$. By (32),

$$\frac{1}{x_i^*}\frac{\partial \pi_i^*}{\partial \gamma_i} = \left(x_i^* - \frac{b_i}{\gamma_i} + \frac{2}{\epsilon}\right)\frac{\left[1 + \frac{\epsilon B_{-i}}{2B_i}\left(x_i^* - \frac{b_i}{\gamma_i}\right)\right]}{\left[1 + \frac{\epsilon B}{2B_i}\left(x_i^* - \frac{b_i}{\gamma_i}\right)\right]} - \frac{2}{\epsilon} - \frac{1}{2}x_i^*. \tag{33}$$

After rearranging terms, (33) implies that $\frac{\partial \pi_i^*}{\partial \gamma_i} < 0$ iff

$$-\frac{1}{2}x_i^* + \frac{\epsilon}{4B_i}\left(x_i^* - \frac{b_i}{\gamma_i}\right)\left[2B_{-i}\left(x_i^* - \frac{b_i}{\gamma_i}\right) - Bx_i^*\right] < 0. \tag{34}$$

Let $m_i = x_i^* - \frac{b_i}{\gamma_i}$ and recall that $A_i = \frac{b_i}{\gamma_i}$. Then, (34) is equivalent to:

$$\epsilon\left(B - 2B_i\right)m_i^2 - \left(\epsilon B A_i + 2B_i\right)m_i - 2B_i A_i < 0. \tag{35}$$

Clearly, if $\frac{B_i}{B} \geq \frac{1}{2}$, then (35) holds and $\frac{\partial \pi_i^*}{\partial \gamma_i} < 0$. In contrast, if $\frac{B_i}{B} < \frac{1}{2}$, then (35) does not hold if m_i is sufficiently large. We note the following: By proposition 1, and by the fact that $\frac{\partial x^*}{\partial R_0} > 0$ (by Lemma 1), it is easy to see that x_i^* is increasing indefinitely in R_0. Since B_i, A_i, B and ϵ are all positive, there is an $\widehat{m}_i > 0$ such that (35) holds iff $m_i < \widehat{m}_i$. Thus, $\frac{\partial \pi_i^*}{\partial \gamma_i} < 0$ if $\frac{B_i}{B} < \frac{1}{2}$ and $m_i < \widehat{m}_i$. Let $\widehat{R}_0$ be the solution to $m_i = \widehat{m}_i$ or $x_i^* = \widehat{m}_i + \frac{b_i}{\gamma_i}$. Since for $R_0 = 0$ we have $x_i^* = \frac{b_i}{\gamma_i}$ and since x_i^* is increasing indefinitely in R_0, there exists a unique $\widehat{R}_0$ such that $m_i = \widehat{m}_i$. In this case, $x_i^* = \widehat{m}_i + \frac{b_i}{\gamma_i}$. We conclude that if $B_i < \frac{1}{2}B$, then $\frac{\partial \pi_i^*}{\partial \gamma_i} < 0$ whenever $R_0 < \widehat{R}_0$ and $\frac{\partial \pi_i^*}{\partial \gamma_i} > 0$ if $R_0 > \widehat{R}_0$. Finally, by Proposition 1, $\frac{\partial \pi_i^*}{\partial \gamma_j} = -\gamma_i\left(\frac{2}{\epsilon} + \frac{B_i}{B}\left(x^* - A\right)\right)\frac{\partial x_i^*}{\partial \gamma_j}$.Since $\frac{\partial x_i^*}{\partial \gamma_j} > 0$, it follows that $\frac{\partial \pi_i^*}{\partial \gamma_j} < 0$, as claimed.■

Proof of Proposition 7. Following the first part of the proof of Proposition 1, the allocations $(x_i)_{i\in N_0}$ and $(y_i)_{i\in N}$ that maximize Π are given by:

$$y_i = \frac{P_i R_0^{0.5}\exp\left(-\frac{\epsilon}{2}x\right)}{C^{0.5}},\ i \in N \tag{36}$$

where $x = \sum_{i\in N_0} x_i$ and $C = \sum_{i\in N}\frac{P_i}{v_i}$. Let $P = \sum_{i\in N_0} P_i$. Then, by (12) and (36)

$$\Pi = \sum_{i\in N_0}\left[b_i x_i - \frac{1}{2}\gamma_i x_i^2\right] - \frac{2PR_0^{0.5}\exp\left(-\frac{\epsilon}{2}x\right)}{C^{0.5}}$$

Thus, for $i \in N_0$

$$\frac{\partial \Pi}{\partial x_i} = 0 \text{ iff } x_i = \frac{b_i}{\gamma_i} + \frac{\epsilon P R_0^{0.5}\exp\left(-\frac{\epsilon}{2}x\right)}{C^{0.5}\gamma_i} \tag{37}$$

Therefore,

$$x = \sum_{i\in N_0} x_i = \sum_{i\in N_0}\frac{b_i}{\gamma_i} + \frac{\epsilon W R_0^{0.5}\exp\left(-\frac{\epsilon}{2}x\right)}{C^{0.5}} \tag{38}$$

where $W = \Gamma P$ and $\Gamma = \sum_{j\in N_0}\frac{1}{\gamma_j}$. Let $x(W)$ be the solution of (38). Note that the non-cooperative solution x^* satisfies (see Proposition 1) $x^* = \sum_{i\in N_0}\frac{b_i}{\gamma_i} + \frac{\epsilon B R_0^{0.5}}{C^{0.5}}\exp\left(-\frac{\epsilon}{2}x^*\right) = x(B)$, where $B = \sum_{i\in N_0}\frac{P_i}{\gamma_i}$. Thus, to prove that $x^* < x$, we need to show that $x(B) < x(W)$. Equivalently, $B\exp\left(-\frac{\epsilon}{2}x(B)\right) < W\exp\left(-\frac{\epsilon}{2}x(W)\right)$. Since

$$W = \Gamma P = \left(\sum_{i\in N_0}\frac{1}{\gamma_i}\right)\left(\sum_{i\in N_0}P_i\right) > \sum_{i\in N_0}\frac{P_i}{\gamma_i} = B$$

it is sufficient to prove that $f(W) = W\exp\left(-\frac{\epsilon}{2}x(W)\right)$ is increasing in W. This is shown by the following Claim.

Claim. The solution x of (38) is increasing in W, namely,

$$\frac{\partial x}{\partial W}=\frac{\epsilon R_0^{0.5}}{C^{0.5}}\exp\left(-\frac{\epsilon}{2}x\right)/\left[1+\frac{\epsilon^2 W R_0^{0.5}}{2C^{0.5}}\exp\left(-\frac{\epsilon}{2}x\right)\right]>0$$

Proof. The proof of the claim follows directly from (38).
Next, let $a=\exp\left(-\frac{\epsilon}{2}x(W)\right)$. By the claim,

$$\begin{aligned}\frac{\partial f(W)}{\partial W} &= a-\frac{\epsilon}{2}Wa\frac{\partial x}{\partial W}\\ &= a-\frac{\epsilon^2 R_0^{0.5}Wa^2}{2C^{0.5}}/\left[1+\frac{\epsilon^2 R_0^{0.5}Wa}{2C^{0.5}}\right]\\ &= \frac{a}{1+\frac{\epsilon^2 R_0^{0.5}Wa}{2C^{0.5}}}>0\end{aligned}$$

Consequently, $x^*<x$. Further, by (36) $y_i<y_i^*$ for all $i\in N$. Finally, by (37) and by Proposition 1, $x_i^*=x_i(P_i)$. Hence, $x_i^*<x_i$ iff $x_i(P_i)<x_i(P)$ or

$$P_i\exp\left(-\frac{\epsilon}{2}x(P_i)\right)<P\exp\left(-\frac{\epsilon}{2}x(P)\right)$$

But this follows from the fact that $P_i<P$ and the fact that $\frac{\partial f(W)}{\partial W}>0$.■

Proof of Proposition 8. Let $N_0\subseteq N$ and let $\widetilde{x}_i(N_0)$ be the solution of the first order equilibrium conditions, namely,

$$\widetilde{x}_i(N_0)=A_i+\frac{B_i\left[\widetilde{x}(N_0)-A(N_0)\right]}{B(N_0)} \tag{39}$$

where $\widetilde{x}(N_0)$ is the unique solution x of

$$x=A(N_0)+\frac{\epsilon B(N_0)R_0^{0.5}\exp\left(-\frac{\epsilon}{2}x\right)}{C(N)^{0.5}} \tag{40}$$

That is, if we restrict the set of cooperating countries to N_0, then (39) and (40) describe the unconstrained equilibrium actions of the countries in N_0. Clearly, if $\widetilde{x}_i(N_0)\geq 0$ for all $i\in N_0$, then $x_i^*(N_0)=\widetilde{x}_i(N_0)$ for all $i\in N_0$, but $\widetilde{x}_i(N_0)$ may be negative if $b_i<0$. We make use of the following Lemma.

Lemma 5. *Let $N_0\neq\phi$ be a subset of N and let $k\in N\backslash N_0$ and $N_0'=N_0+k$. Suppose that $\widetilde{x}_k(N_0')\geq 0$. Then, for all $i\in N_0$, (1) $\widetilde{x}(N_0')\geq\widetilde{x}(N_0)$. (2) $\widetilde{x}_i(N_0')\leq\widetilde{x}_i(N_0)$ and $\widetilde{y}_i(N_0')\leq\widetilde{y}_i(N_0)$. (3) $\widetilde{\lambda}_i(N_0')\leq\widetilde{\lambda}_i(N_0)$,* $\widetilde{\pi}_i(N_0')\geq\widetilde{\pi}_i(N_0)$ and $\widetilde{\pi}_T(N_0')\leq\widetilde{\pi}_T(N_0)$. (4) All the inequalities above are strict if $\widetilde{x}_k(N_0')>0$. (5) All the inequalities in (1)-(3) are reversed if $\widetilde{x}_k(N_0')\leq 0$ and they in addition are strict if $\widetilde{x}_k(N_0')<0$.

Proof of Lemma 5. By the first order conditions that determine the proactive effort, $\widetilde{x}_k(N_0')\geq 0$ iff

$$A_k\geq-\frac{B_k\epsilon R_0^{0.5}\exp\left(-\frac{\epsilon}{2}\widetilde{x}(N_0')\right)}{C(N)^{0.5}}.$$

In this case,

$$\widetilde{x}(N_0') \geq A(N_0)+\frac{\epsilon B(N_0)R_0^{0.5}\exp\left(-\frac{\epsilon}{2}\widetilde{x}(N_0')\right)}{C(N)^{0.5}} \text{ and} \tag{41}$$

$$\widetilde{x}(N_0) = A(N_0)+\frac{\epsilon B(N_0)R_0^{0.5}\exp\left(-\frac{\epsilon}{2}\widetilde{x}(N_0)\right)}{C(N)^{0.5}} \tag{42}$$

By (41) and (42), $\widetilde{x}(N_0') \geq \widetilde{x}(N_0)$ and strict inequality holds if $\widetilde{x}_k(N_0') > 0$. Note that all inequalities are reversed if $\widetilde{x}_k(N_0') \leq 0$ and they are strict if $\widetilde{x}_k(N_0') < 0$. Since $\widetilde{x}_k(N_0') \geq 0$, there exists some b_k' with $b_k' < b_k$ such that $\widetilde{x}_k(N_0') = 0$. In this case, $\widetilde{x}(N_0') = \widetilde{x}(N_0)$ and $\widetilde{x}_i(N_0') = \widetilde{x}_i(N_0)$ for all $i \in N_0$. By proposition 2, $\frac{\partial \widetilde{x}_i(N_0')}{\partial b_k} < 0$ (the proof of this claim holds true for all values of b_k irrespective of wether it is negative or not). Thus, increasing the benefit of cooperation for k from b_k' to b_k decreases $\widetilde{x}_i(N_0')$, as claimed. Further, since $\widetilde{y}_i(N_0) = \frac{\gamma_i \widetilde{x}_i(N_0) - b_i}{\epsilon}$ and $\widetilde{y}_i(N_0') = \frac{\gamma_i \widetilde{x}_i(N_0') - b_i}{\epsilon}$ for $i \in N_0$, we have that $\widetilde{y}_i(N_0) \leq \widetilde{y}_i(N_0')$. The same proof fits the case $\widetilde{x}_k(N_0') \leq 0$ except that, for this case, $b_k\acute{} \geq b_k$ and the results are reserved as well.

By Proposition 1, $\widetilde{\lambda}_i(N_0') = \frac{P_i R_0^{0.5}}{v_i C(N)^{0.5}} \exp\left(-\frac{\epsilon}{2}\widetilde{x}(N_0')\right)$. Since $\widetilde{x}(N_0') \geq \widetilde{x}(N_0)$, we have $\widetilde{\lambda}_i(N_0') \leq \widetilde{\lambda}_i(N_0)$. To show that $\widetilde{\pi}_i(N_0') > \widetilde{\pi}_i(N_0)$ for all $i \in N_0$, we let $\Delta = \widetilde{\pi}_i(N_0') - \widetilde{\pi}_i(N_0)$. By Proposition 1,

$$\begin{aligned}
\Delta &= b_i[\widetilde{x}_i\left(N_0'\right) - \widetilde{x}_i(N_0)] - \frac{2\gamma_i}{\epsilon}[\widetilde{x}_i\left(N_0'\right) - \widetilde{x}_i(N_0)] \\
&\quad -\frac{1}{2}\gamma_i[\widetilde{x}_i\left(N_0'\right) - \widetilde{x}_i(N_0)][\widetilde{x}_i\left(N_0'\right) + \widetilde{x}_i(N_0)] \\
&= [\widetilde{x}_i\left(N_0'\right) - \widetilde{x}_i(N_0)][b_i - \frac{2\gamma_i}{\epsilon} - \frac{1}{2}\gamma_i[\widetilde{x}_i\left(N_0'\right) + \widetilde{x}_i(N_0)]]
\end{aligned}$$

By Proposition 1, $\widetilde{x}_i(N_0') \geq \frac{b_i}{\gamma_i}$ and $\widetilde{x}_i(N_0) \geq \frac{b_i}{\gamma_i}$. Since $\widetilde{x}_i(N_0') \leq \widetilde{x}_i(N_0)$, it follows that $\Delta \geq -\frac{2\gamma_i}{\epsilon}[\widetilde{x}_i(N_0') - \widetilde{x}_i(N_0)] \geq 0$. Finally, by Proposition 1, $\widetilde{\pi}_T(N_0') \leq \widetilde{\pi}_T(N_0)$ iff $\exp\left(-\frac{\epsilon}{2}\widetilde{x}(N_0')\right) \leq \exp\left(-\frac{\epsilon}{2}\widetilde{x}(N_0)\right)$, which certainly holds, since $\widetilde{x}(N_0') \geq \widetilde{x}(N_0)$.■

Next, we prove Proposition 8 in a constructive way using Lemmas 6 and 7 below.

Lemma 6. Let $N_0 \subset N$, $i \in N_0$ and $k \notin N_0$. (1) If $\widetilde{x}_k(N_0 + k) \geq 0$ and $\widetilde{x}_i(N_0 + k) \leq 0$ and at least one inequality is strict, then $\frac{b_k}{P_k} > \frac{b_i}{P_i}$; (2) if $\widetilde{x}_k(N_0 + k) \leq 0$ and $\widetilde{x}_i(N_0 + k) \geq 0$ and at least one inequality is strict, then $\frac{b_k}{P_k} < \frac{b_i}{P_i}$.

Proof of Lemma 6. By the equilibrium first order conditions,

$$\begin{aligned}
\widetilde{x}_i(N_0 + k) &= A_i + \frac{B_i\left[\widetilde{x}(N_0 + k) - A(N_0 + k)\right]}{B(N_0 + k)} \leq 0 \\
\widetilde{x}_k(N_0 + k) &= A_k + \frac{B_k\left[\widetilde{x}(N_0 + k) - A(N_0 + k)\right]}{B(N_0 + k)} > 0
\end{aligned}$$

Therefore, $-\frac{A_i}{B_i} = -\frac{b_i}{P_i} \geq \frac{[\widetilde{x}(N_0+k) - A(N_0+k)]}{B(N_0+k)} > -\frac{A_k}{B_k} = -\frac{b_k}{P_k}$. The proof of part (2) is similar.■

Lemma 7. Suppose that $\frac{b_1}{P_1} \geq \frac{b_2}{P_2} \geq ... \geq \frac{b_n}{P_n}$. Let $1 < m < k \leq n$. If $\widetilde{x}_m(1, 2, ..., m) \leq 0$, then $\widetilde{x}_k(1, 2, ..., m-1, k) \leq 0$.

Proof of Lemma 7. Claim**.** $\widetilde{x}_k(1, 2, ..., m, k) \leq 0$. Otherwise, if $\widetilde{x}_k(1, 2, ..., m, k) > 0$, then by part (2) of Lemma 6, $\widetilde{x}_m(1, 2, ..., m, k) < \widetilde{x}_k(1, 2, ..., m) \leq 0$. By Lemma 7, $\frac{b_k}{P_k} > \frac{b_m}{P_m}$, a contradiction. Suppose to the contrary that $\widetilde{x}_k(1, 2, ..., m-1, k) > 0$. Then, $\widetilde{x}_m(1, 2, ..., m, k) > 0$. Otherwise, by part (5) of Lemma 5, $0 \geq \widetilde{x}_k(1, 2, ..., m, k) \geq \widetilde{x}_k(1, 2, ..., m-1, k)$, a contradiction. Applying again part (2) of Lemma 5, we have that $\widetilde{x}_k(1, 2, ..., m, k) \geq \widetilde{x}_k(1, 2, ..., m-1, k) > 0$ and this contradicts the claim.■

We are now ready to complete the proof of Proposition 8.

1. A non constructive proof for the existence of a pure strategy subgame perfect equilibrium is straightforward. Indeed, the proof of Proposition 1 implies that for all $(x_1, ..., x_n)$ the second stage of the game has a unique pure strategy equilibrium $((R_1^*, ..., R_n^*), (y_1^*, ..., y_n^*))$, where R_i^* and y_i^* are all functions of $(x_1^*, ..., x_n^*)$. Substituting R_i^* and y_i^* in the payoff function π_i we obtain a payoff function which is strictly concave in x_i. In addition, it can be assumed without loss of generality that:

$$0 \leq x_i \leq \frac{b_i}{\gamma_i} + \frac{\epsilon P_i}{\gamma_i} \left(\frac{R_0}{\sum_{k=1}^{n} \frac{P_k}{v_k}} \right)^{0.5}$$

It is well known that concave payoff functions defined on compact strategy sets guarantee the existence of a pure strategy equilibrium. We provide next a constructive proof.

Without loss of generality assume that $\frac{b_1}{P_1} \geq \frac{b_2}{P_2} \geq ... \geq \frac{b_n}{P_n}$.*First.* Let $N_{0,1} = \{1\}$. If $\widetilde{x}_2(1,2) \leq 0$, then by Lemma 7 $\widetilde{x}_k(1,k) \leq 0$ for all $1 < k \leq n$ and $N_{0,1}$ is a candidate for the equilibrium set of cooperating countries (recall that $b_1 = 0$ if $|N_0| = 1$ and hence $\widetilde{x}_1(1) = x_1^*(1) > 0$). Otherwise, let $N_{0,2} = \{1,2\}$. By Lemma 6, $\widetilde{x}_1(1,2) > 0$. If $\widetilde{x}_3(1,2,3) \leq 0$, then by Lemma 7 $N_{0,2}$ is sustainable. Otherwise, if $\widetilde{x}_3(1,2,3) > 0$, then by Lemma 6, $\widetilde{x}_2(1,2,3) > 0$ and $\widetilde{x}_1(1,2,3) > 0$. If there is m, $1 \leq m \leq n$ such that $\widetilde{x}_m(1,2,...,m) > 0$ and $\widetilde{x}_{m+1}(1,2,...,m+1) \leq 0$, the sustainable set is $\{1,2,...,m\}$. Otherwise it is N.

Suppose that the process ends after n steps and the candidate is $N_{0,m}$, $1 \leq m \leq n$. Consider the following first period choices of the countries in N: $x_i = \widetilde{x}_i(N_{0,m})$, $i = 1,..,m$ and $x_i = 0$, $i = m+1,...,n$. Since $\widetilde{x}_i(N_{0,m}) > 0$ for all $i \in N_{0,m}$, then $x_i^*(N_{0,m}) = \widetilde{x}_i(N_{0,m})$ for $i \in N_{0,m}$ and $x_i^* = 0$ for $i \in N \backslash N_{0,m}$. Clearly, no country has an incentive to deviate and $N_{0,m}$ is sustainable.

2. Suppose to the contrary that ϕ is sustainable. Since $b_i = 0$ for $N_0 = \{i\}$, then $x_i^*(N_0) > 0$ and i is better off deviating from ϕ to $N_0 = \{i\}$.

3. Suppose that N_0 is an equilibrium set of cooperating countries and suppose that $b_i \geq 0$ while $i \notin N_0$. The first order equilibrium condition for $i \in N_0 + i$ is

$$x_i = \frac{b_i}{\gamma_i} + \epsilon B_i \left(\frac{R_0}{C} \right)^{0.5} \exp \left(-\frac{\epsilon}{2} \sum_{k \in N_0 + i} x_k \right) > 0$$

and hence i has an incentive to join N_0, a contradiction.■