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Capital adjustment costs and firm risk aversion

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Abstract

We show that the equilibrium of a standard real business cycle model with capital adjustment costs can be supported as an equilibrium with a risk averse firm and no adjustment costs. Our result sheds light on the implications of a utility maximizing (UM) firm, which is one of the approaches proposed in the literature to get around the firm objective problem when markets are incomplete.

Keywords: Adjustment costs; RBC model; Analytical solution

JEL classification: C68; E13

1. Introduction

Capital adjustment costs have been extensively used in the literature. In particular, several authors, like Lettau (1998), Jermann (1998) and Danthine and Donaldson (2002), have shown that the presence of installation costs can help to improve the asset pricing implications of the standard real business cycle model, specially when combined with other frictions. In the present work, we show that the equilibrium of the model in the presence of capital adjustment costs and a value maximizing (VM) firm can also be supported as an equilibrium with a firm that maximizes the expected utility of its profits and is risk averse, while it faces no adjustment costs. Although our result is local, in the sense that we rely on approximations of the model around the steady state, we perform a numerical exercise, suggesting that the equivalence also holds globally.

We believe that our result is important for two reasons. First, our approach has interest in itself, since it can be used to compare the equilibrium properties of different economies as long as they can

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be approximated in log-linear form. In essence, it consists of approximating the equilibrium law of motion for the endogenous variables in each economy as a function of the model parameters. While obtaining the solution to each model involves choosing a set of parameters, we endogenously derive the parameter values that yield the same allocation in the different economies by solving a non-linear system of equations.

Apart from this, a utility maximizing (UM) firm objective can be used in a context with shareholder heterogeneity, infinitely lived firms, and incomplete financial markets, in which case profit maximization is no longer well defined. This is indeed one of the approaches that have been proposed in the literature to get around the firm objective problem by authors like Radner (1972), Sandmo (1972), Sondermann (1974) and Leland (1972). While the previous literature uses a static context, we incorporate such an objective into a dynamic setting, shedding some light on its equilibrium implications.

The paper is organized as follows. The following section presents the model, and Section 3 establishes the equivalence result. Finally, Section 4 briefly concludes.

2. The model

We consider a decentralized version of the standard real business cycle model, populated by a large number of identical infinitely lived firms and households.

2.1. Households

Each period, the representative household maximizes his expected lifetime utility subject to a sequential budget constraint:

$$\text{Max}_{\{C_t, A_{t+1}, L_t^s\}} E_t \sum_{j=0}^{\infty} \beta^j U(C_{t+j}) \quad \text{where } U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \tag{1}$$

s.t.

$$C_t + A_t' P_t^a = A_{t-1}'(P_t^a + D_t^a) + W_t L_t^s \tag{2}$$

where γ and β represent the household risk aversion and time discount factor, respectively. A_t is a vector of financial assets, and P_t^a and D_t^a represent the vectors of asset prices and dividends. The asset vector contains equity shares of the representative firm, whose outstanding number is assumed to be equal to one, and possibly other assets, assumed to be in zero net supply. Apart from their asset income, households receive labor income, equal to the aggregate wage rate W_t times their labor supply L_t^s. Each household is endowed with one unit of time, which he can allocate to leisure or to productive labor L_t^s. Given that leisure does not enter the utility function, however, the entire time endowment is allocated to labor. The first order conditions for the problem above lead to the usual Euler equations determining asset prices:

$$P_t^a = E_t[M_t^{t+1}(P_{t+1}^a + D_{t+1}^a)] \quad \text{where } M_t^{t+j} = \beta^j \left(\frac{C_{t+j}}{C_t}\right)^{-\gamma} \tag{3}$$

2.2. Firms

Each period, the representative firm combines aggregate capital K_{t-1} and labor L_t to produce a single good Y_t according to the following constant returns to scale technology:

$$Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{4}$$

where Z_t is a random productivity shock assumed to follow the stationary process:

$$\log Z_t = \psi_z \log Z_{t-1} + \varepsilon_t^z, \quad \varepsilon_t^z \sim N(0, \sigma_z^2) \tag{5}$$

Investment I_t is entirely financed by retained earnings or gross profits $X_t = Y_t - W_t L_t$, and the residual of gross profits and investment is paid out as dividends to the firm owners. Capital accumulates according to the following equation:

$$K_t = g\left(\frac{I_t}{K_{t-1}}\right)K_{t-1} + (1-\delta)K_{t-1} \tag{6}$$

where δ is the depreciation rate, and g is a positive, concave installation function. If $g(I_t/K_{t-1}) = (I_t/K_{t-1})$, there are no capital installation costs. Each period, the representative firm chooses capital K_t and labor L_t to maximize, subject to the previous constraints, the expected lifetime utility of its nets cash flow $N_t = X_t - I_t$:

$$\text{Max}_{\{K_t, L_t\}} E_t \sum_{j=0}^{\infty} \phi_j U_f(N_{t+j}) \quad \text{where} \quad U_f(N_{t+j}) = \frac{N_{t+j}^{1-\gamma_f}}{1-\gamma_f} \tag{7}$$

where ϕ and γ_f are the firm discount factor and firm risk aversion coefficient, respectively. If we impose the labor market clearing condition, implying that $L_t = 1$, the first order conditions for the problem above are given by:

$$W_t = (1-\alpha)Y_t \tag{8}$$

$$\frac{1}{g'(I_t/K_{t-1})} = E_t\left[\phi_f \frac{N_{t+1}^{-\gamma_f}}{N_t^{-\gamma_f}}\left\{\alpha Z_{t+1} K_t^{\alpha-1} + \frac{(1-\delta) + g(I_{t+1}/K_t)}{g'(I_{t+1}/K_t)} - \frac{I_{t+1}}{K_t}\right\}\right] \tag{9}$$

3. Value maximizing versus risk averse firms

We consider two different objectives for the firm. The first, denoted by VM firm objective, assumes that $\gamma_f = 0$, while $\phi_j = M_t^{t+j}$. Note that this corresponds to the usual assumption that the firm maximizes its market value, equal to the present discounted value of its future net cash flows:

$$E_t \sum_{j=0}^{\infty} M_t^{t+j}(X_{t+j} - I_{t+j}) = E_t \sum_{j=0}^{\infty} M_t^{t+j} N_{t+j} \tag{10}$$

The second objective, denoted by UM firm objective, assumes that $\gamma_f > 0$, implying that the firm is risk averse, while the firm discount factor is equal to some number β_f, with $\phi_j = \beta_f^j$. The relationship between the two firm objectives is summarized by the following proposition.

Proposition 1. *Assume that an economy with no capital adjustment costs is populated by a representative household with constant relative risk aversion (CRRA) utility and a risk averse firm with CRRA utility U_f and the same time discount factor β_f as the household. Assume that there is an otherwise identical economy, in which the firm is VM and is subject to capital installation costs of the form $g(I_t/K_{t-1})$, with the properties that $g(I_s/K_s) = (I_s/K_s)$ and $g'(I_s/K_s) = 1$, where I_s/K_s is the steady state investment to capital ratio. Then, it is possible to find parameters determining the level of firm risk aversion in the first economy and the level of capital adjustment costs in the second, such that both economies have the same equilibrium behavior around the steady state.*

Proof. Intuitively, note that a risk averse firm will try to smooth its net cash flow over time, implying that it will choose a lower (higher) investment level after a good (bad) shock. Thus, investment will be smoother, and consumption more volatile, if the firm is risk averse, as in the presence of capital installation costs. To prove this equivalence, we can define the steady state elasticity $\zeta \in [0, 1)$:

$$\zeta = -\frac{g''(I_s/K_s)(I_s/K_s)}{g'(I_s/K_s)} \tag{11}$$

which is the parameter determining the degree of capital adjustment costs in the second economy. In particular, if ζ is equal to zero, there are no installation costs. Further, following Campbell (1994), we can find an approximate closed form solution for the law of motion of the endogenous variables in each economy by replacing the true constraints and Euler equations with log-linear approximations. In particular, if we denote by η_{Xx} the elasticity of variable X with respect to variable x, the elasticities of logged output, capital, investment, and net cash flows with respect to the two logged state variables, $z = \log(Z)$ and $k = \log(K)$, are given by:

$$\eta^j_{yk} = \alpha \quad \text{and} \quad \eta^j_{yz} = 1 \tag{12}$$

$$\eta^j_{kk} = \lambda_1 + \lambda_3 \eta^j_{ck} \quad \text{and} \quad \eta^j_{kz} = \lambda_2 + \lambda_3 \eta^j_{cz} \tag{13}$$

$$\eta^j_{ik} = \lambda_4 \alpha - \lambda_5 \eta^j_{ck} \quad \text{and} \quad \eta^j_{iz} = \lambda_4 - \lambda_5 \eta^j_{cz} \tag{14}$$

$$\eta^j_{nk} = \lambda_6 \alpha - \lambda_7 \eta^j_{ik} \quad \text{and} \quad \eta^j_{nz} = \lambda_6 - \lambda_7 \eta^j_{iz} \tag{15}$$

where $j = 1, 2$ indexes the economy, and the coefficients denoted by λ are constants that depend on vector of model parameters $\Phi = (\alpha, \beta, \delta, \Psi_z)$. Using the equilibrium system of equations before the log-linearization, it is easy to see that the assumptions on U_f, β_f, and g required by the proposition lead to the same non-stochastic steady state in the two economies. Further, the previous equations show that the equilibrium properties around the steady state will also be identical if the two consumption elasticities, η^j_{ck} and η^j_{cz}, are the same for $j = 1, 2$. Using the method of undetermined coefficients, it can be shown that these elasticities are the solutions to the following equations:

$$\eta^{j}_{ck} = \frac{1}{Q_{2j}}\left\{ -Q_{1j} - \sqrt{Q^2_{1j} - 4Q_{0j}Q_{2j}} \right\} \quad \text{for } j = 1, 2 \tag{16}$$

$$\eta^{1}_{cz} = f_1(\Phi, \gamma_f, \eta_{ck_1}) \tag{17}$$

$$\eta^{2}_{cz} = f_2(\Phi, \zeta, \gamma, \eta_{ck_2}) \tag{18}$$

where the constants denoted by Q depend from Φ and γ_f in the first economy, and from Φ, ζ and γ in the second, while f_1 and f_2 are two non-linear functions. Clearly, for a given parameter vector $\Theta = (\Phi, \gamma)$, assumed to be the same in the two economies, equivalence will obtain if there exist a vector (ζ^*, γ_f^*) that solves the following equation system:

$$\eta^{2}_{ck}(\Theta, \zeta^*) - \eta^{1}_{ck}(\Phi, \gamma_f^*) = 0 \tag{19}$$

$$\eta^{1}_{cz}(\Theta, \zeta^*) - \eta^{1}_{cz}(\Phi, \gamma_f^*) = 0 \tag{20}$$

In our relatively simple setup, the system can be rewritten as follows:

$$F(\Theta, \gamma_f(\zeta^*), \zeta^*) = 0 \tag{21}$$

where F is a continuous function that results by expressing γ_f as a function of ζ from the first equation, and by substituting it in the second. Given this, for a particular vector Θ, the existence of a solution $\zeta^*(\Theta) \in (0, 1)$ is guaranteed if (i) $\lim_{\zeta \to 0} F(\Theta, \gamma_f(\zeta), \zeta)$ and $\lim_{\zeta \to 1} F(\Theta, \gamma_f(\zeta), \zeta)$ have opposite sign, while it is considered to be relevant if (ii) $\gamma_f(\zeta^*(\Theta)) \in (0, \infty)$. Conditions (i) and (ii) are satisfied for a large number of parameter values, and, in particular, with $\gamma = 1$ and $\alpha = 0.36$, they hold for the usual parameter ranges in the real business cycle literature, i.e. $\beta \in [0.96, 0.99]$, $\delta \in (0, 0.1]$ and $\psi_z \in [0.95, 1)$.

4. Global equivalence

To investigate if the previous equivalence also holds globally, we have obtained the solution ζ^* and γ_f^* from the previous equation system, and have numerically solved each of the two economies with these values using the Parameterized Expectations approach, which does not rely on approximations around the steady state[1]. We have chosen $\Phi = (\alpha, \beta, \delta, \psi_z) = (0.36, 0.99, 0.025, 0.95)$ following the literature with quarterly data, while γ has been set to 1 in both economies. Further, the adjustment cost function has been specified as $g(I_t/K_{t-1}) = (b/1-\zeta)(I_t/K_{t-1})^{1-\zeta} + c$, where b and c have been chosen to satisfy the conditions in the proposition. With these parameter values, the solution to the previous system is given by $(\zeta^*, \gamma_f^*) = (0.565, 1.44)$. The similarities between the impulse response

[1] See Marcet and Lorenzoni (1998) for details of the numerical algorithm.

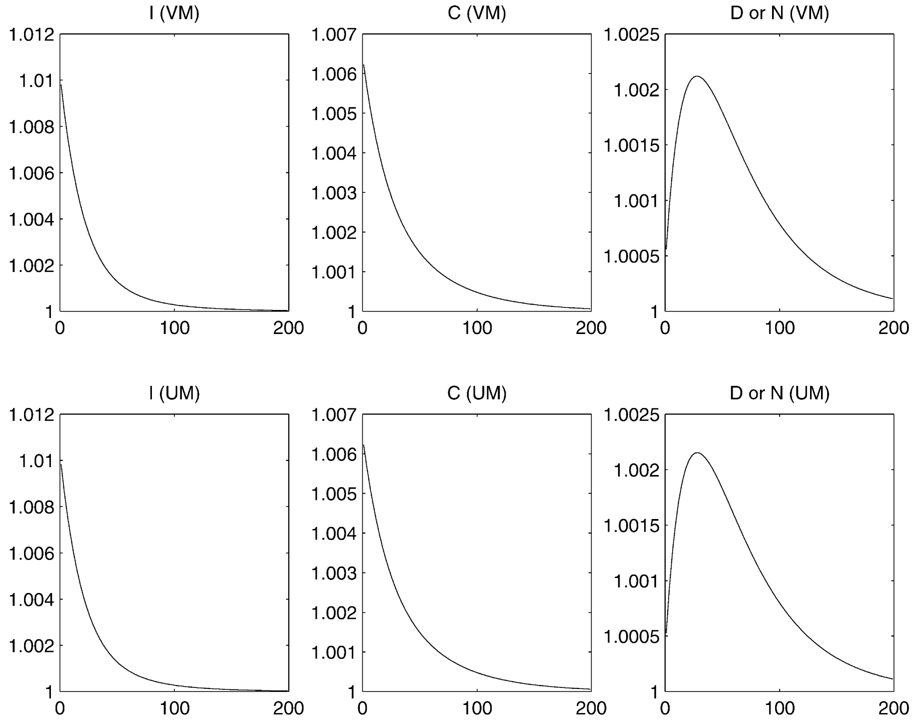

Fig. 1. Impulse responses for Economy 1 (UM) and 2 (VM).

functions from the numerical solutions, depicted in Fig. 1 for some of the variables, suggest that the equivalence also holds globally.

5. Conclusions

Using a method that relies on log-linear approximations around the steady state, we have shown that a risk averse firm maximizing the expected utility of profits in a representative agent real business cycle model is equivalent to the presence of capital installation costs and the usual VM firm. Intuitively, if the firm is risk averse, it will smooth its net cash flows over time through a less elastic investment, leading in turn to a more variable consumption pattern. The numerical solution obtained with a non-linear method also indicates that the equivalence also holds outside the steady state region. The equivalence result is important, since it sheds light on the equilibrium implications of a UM firm, which can be used in a context with household heterogeneity to get around the firm objective problem when markets are incomplete[2].

[2]This is done, for example, in Carceles-Poveda (2001a,b).

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