

Technical Supplement to:
**“Pre-negotiation Public Commitment in Domestic and
International Bargaining”**

Bahar Leventoğlu
Department of Political Science
Stony Brook University
Stony Brook, NY 11794
e-mail: bahar.leventoglu@stonybrook.edu

Ahmer Tarar
Department of Political Science
Texas A&M University
4348 TAMU
College Station, TX 77843-4348
e-mail: ahmertarar@polisci.tamu.edu

February 3, 2005

1 Introduction

In this technical supplement, we first provide a proof that the equilibrium described in the paper is the unique stationary subgame-perfect equilibrium (in the paper, we prove that it is an equilibrium, but do not provide the much longer proof of uniqueness). We then construct and analyze a model that demonstrates that the citizens will rationally choose to impose audience costs on their leader, a claim made in the conclusion of the paper.

2 Proof of Proposition 1

An equilibrium is a *stationary pure strategy equilibrium* if it is a pure strategy equilibrium, and the equilibrium of the bargaining subgame is stationary. We will prove the existence and uniqueness of the stationary pure strategy equilibrium in this technical supplement.

Suppose that 1 commits to a and 2 commits to b . In a stationary subgame-perfect equilibrium (SPE) of the bargaining subgame, negotiator 1 proposes $(x, 1 - x)$ and negotiator 2 accepts it. Negotiator 2 proposes $(1 - y, y)$ and negotiator 1 accepts it. For negotiator 2 to accept 1's proposal, negotiator 2's overall payoff for accepting it should be (at least) equal to his overall payoff if he rejects negotiator 1's proposal, makes a counter proposal himself and negotiator 1 accepts it. Moreover, for negotiator 1 to accept 2's proposal, negotiator 1's overall payoff if she accepts 2's proposal should be (at least) equal to her overall payoff if she rejects 2's proposal, makes a counter proposal herself and negotiator 2 accepts it. That is:

$$(1 - x) - C_2(1 - x, b) = \delta[y - C_2(y, b)] \tag{1}$$

$$(1 - y) - C_1(1 - y, a) = \delta[x - C_1(x, a)] \tag{2}$$

Furthermore, 1 makes the first offer with probability p . In this case 1's utility level is given by $x - C_1(x, a)$. With probability $1 - p$, 2 makes the first offer. In this case, 1's utility level is given by $(1 - y) - C_1(1 - y, a) = \delta[x - C_1(x, a)]$. Therefore, 1's expected utility in the equilibrium of the bargaining subgame can be calculated as

$$V_1(a, b) = [p + (1 - p)\delta][x - C_1(x, a)].$$

Similarly, 2's expected utility in the equilibrium of the bargaining subgame can be calculated as

$$V_2(a, b) = [p\delta + (1 - p)][y - C_2(y, b)].$$

Before proceeding with the proof of our main result, we will give four preliminary results.

Lemma A1: For any $a, b \in [0, 1]$, there is a unique solution to the equation system (1) and (2).

Proof: Since $z - C_2(z, b)$ is an increasing function of z and $\delta < 1$, (1) implies that $1 - x < y$ in (1). Also, (1) defines the following relationship between x and y : If $y \geq \frac{b}{\delta} > b$, then $1 - x = \delta y \geq b$; if $\frac{b}{\delta} \geq y \geq b$, then $(1 - x) < b$ and $(1 - x) - \phi_2(b - (1 - x)) = \delta y$; if $y \leq b$, then $(1 - x) < b$ and $(1 - x) - \phi_2(b - (1 - x)) = \delta[y - \phi_2(b - y)]$. Equivalently,

$$y = f(x) = \begin{cases} \frac{1}{\delta} - \frac{x}{\delta}, & \text{if } x \leq 1 - b \\ \frac{1 + \phi_2 - \phi_2 b}{\delta} - \frac{1 + \phi_2}{\delta} x, & \text{if } 1 - b \leq x \leq 1 - \frac{\delta + \phi_2}{1 + \phi_2} b \\ \frac{1 - (1 - \delta)\phi_2 b}{\delta(1 + \phi_2)} - \frac{x}{\delta}, & \text{if } x \geq 1 - \frac{\delta + \phi_2}{1 + \phi_2} b \end{cases} \quad (3)$$

Similarly, (2) implies that $1 - y < x$ in (2). It also defines the following relationship between x and y .

$$y = g(x) = \begin{cases} 1 - \frac{(1 - \delta)\phi_1}{1 + \phi_1} a - \delta x, & \text{if } x \leq a \\ 1 - \frac{\phi_1}{1 + \phi_1} a - \frac{\delta}{1 + \phi_1} x, & \text{if } a \leq x \leq \frac{a}{\delta} \\ 1 - \delta x, & \text{if } x \geq \frac{a}{\delta} \end{cases} \quad (4)$$

The solution to the equation system (1) and (2) is given by the intersection of the curves f and g . Both f and g are decreasing in x . Also, $\frac{1 + \phi_2}{\delta} > \frac{1}{\delta} > \delta > \frac{\delta}{1 + \phi_1}$. That is, f is steeper

than g everywhere. Furthermore, $f(0) > g(0)$ and $f(1) < g(1)$, i.e. f lies above g at $x = 0$ and it lies below g at $x = 1$. Therefore, f and g intersects at a unique point. This implies that there is a unique solution to the equation system (1) and (2). Q.E.D.

Lemma A2: $0 < x < 1$ and $0 < y < 1$ in every equilibrium.

Proof: f lies above g when $x = 0$ or $y = 1$ and it lies below g when $x = 1$ or $y = 0$.

Therefore, f and g intersect at $(x, y) \in (0, 1) \times (0, 1)$. Q.E.D.

Lemma A3: For any $a, b \in [0, 1]$, $x \leq a$ and $1 - y \geq a$ cannot hold in any equilibrium of the bargaining subgame.

Proof: We show in the proof of Lemma A1 that $1 - y < x$. Q.E.D.

Lemma A4: For any $a, b \in [0, 1]$, $y \leq b$ and $1 - x \geq b$ cannot hold in any equilibrium of the bargaining subgame.

Proof: We show in the proof of Lemma A1 that $1 - x < y$. Q.E.D.

Proposition 1 *For any $\phi_1, \phi_2 \geq 0$, the following is the stationary subgame-perfect equilibrium of this game: negotiator 1 makes the public commitment $a^* = \frac{1+\phi_1}{1+\delta+\phi_1+\phi_2}$, negotiator 2 makes the public commitment $b^* = \frac{1+\phi_2}{1+\delta+\phi_1+\phi_2}$, and when they do, in the bargaining subgame the negotiators use the following strategies:*

(a) *Negotiator 1 always proposes $(x^*, 1 - x^*) = (\frac{1+\phi_1}{1+\delta+\phi_1+\phi_2}, \frac{\delta+\phi_2}{1+\delta+\phi_1+\phi_2})$ and always accepts any proposal $(1 - y, y)$ such that $y \leq \frac{1+\phi_2}{1+\delta+\phi_1+\phi_2}$.*

(b) *Negotiator 2 always proposes $(1 - y^*, y^*) = (\frac{\delta+\phi_1}{1+\delta+\phi_1+\phi_2}, \frac{1+\phi_2}{1+\delta+\phi_1+\phi_2})$ and always accepts any proposal $(x, 1 - x)$ such that $x \leq \frac{1+\phi_1}{1+\delta+\phi_1+\phi_2}$.*

Proof of the Proposition:

We will show simultaneously that (i) the strategy profile described in our proposition

constitutes an equilibrium, and (ii) $(x = a, y = b, 1 - y \leq a, 1 - x \leq b)$ must hold in equilibrium.

In equilibrium, $(x \leq a \text{ or } x \geq a)$ and $(1 - y \leq a \text{ or } 1 - y \geq a)$ and $(y \leq b \text{ or } y \geq b)$ and $(1 - x \leq b \text{ or } 1 - x \geq b)$ holds. This gives 16 different potential cases. In the following table, (+) in a column indicates that the corresponding inequality holds; (-) indicates that it does not hold, or in other words the other direction of the inequality holds. Lemmas A3 and A4 imply that the following 7 cases cannot hold in any equilibrium:

$x \leq a$	$1 - y \leq a$	$y \leq b$	$1 - x \leq b$
+	-	+	+
+	-	-	+
+	-	+	-
+	-	-	-
+	+	+	-
-	+	+	-
-	-	+	-

By eliminating all other cases, we will show that $(x = a, y = b, 1 - y \leq a, 1 - x \leq b)$ must hold in equilibrium, which we refer to as Case E. Case E will prove existence and uniqueness.

Case pseudo-E: $(x \leq a, y \leq b, 1 - y \leq a, 1 - x \leq b)$ holds in equilibrium.

In this case, rewriting (1) and (2)

$$(1 - x) - \phi_2(b - (1 - x)) = \delta(y - \phi_2(b - y)) \tag{5}$$

$$(1 - y) - \phi_1(a - (1 - y)) = \delta(x - \phi_1(a - x)) \tag{6}$$

and solving for x and y , we obtain

$$x = \frac{1}{1 + \delta} + \frac{\delta(1 + \phi_2)\phi_1 a - (1 + \phi_1)\phi_2 b}{(1 + \phi_1)(1 + \phi_2)(1 + \delta)} \tag{7}$$

$$y = \frac{1}{1 + \delta} + \frac{\delta(1 + \phi_1)\phi_2 b - (1 + \phi_2)\phi_1 a}{(1 + \phi_1)(1 + \phi_2)(1 + \delta)} \tag{8}$$

Letting $x = a$ and $y = b$, and solving for a and b , we obtain

$$a = a^* = \frac{1 + \phi_1}{1 + \delta + \phi_1 + \phi_2}$$

$$b = b^* = \frac{1 + \phi_2}{1 + \delta + \phi_1 + \phi_2}$$

Since $1 - y < x$ (see the proof of Lemma A3), $x \leq a$ implies $1 - y < a$. Similarly, $y \leq b$ implies $1 - x < b$. Also note that $a^* \in (0, 1)$ and $b^* \in (0, 1)$. Therefore, a^* and b^* are feasible; and given that 1 and 2 commit to a^* and b^* respectively, the stationary equilibrium of the bargaining subgame is given by $x^* = a^*$ and $y^* = b^*$.

Now, we will show that $a = a^*$ and $b = b^*$ constitute an equilibrium of the whole game.

Using (7), 1's payoff from x in Case pseudo-E can be calculated as

$$x - \phi_1(a - x) = \frac{1 + \phi_1}{1 + \delta} - \frac{(1 + \phi_2)\phi_1 a + (1 + \phi_1)\phi_2 b}{(1 + \phi_2)(1 + \delta)}.$$

First, we will show that 1 has no incentive to increase a under Case pseudo-E. Consider an increase in a . Note that $1 - x \leq b$ and $1 - y \leq a$ are slack. Since x in (7) is increasing in a , and (8) implies $\frac{\partial(1-y)}{\partial a} = \frac{\phi_1}{(1+\phi_1)(1+\delta)} < 1$, an increase in a further relaxes these two constraints. y in (8) is decreasing in a , therefore $y \leq b$ relaxes as a increases, as well. Also, $\frac{\partial x}{\partial a} = \frac{\delta\phi_1}{(1+\phi_1)(1+\delta)} < 1$, so that $x \leq a$ becomes slack. Therefore, (7) and (8) continue to solve for x and y in the equilibrium of the bargaining subgame after an increase in a . By Lemma A1, the solution is unique. Since, $V_1(a, b) = [p + (1 - p)\delta][x - \phi_1(a - x)]$ is decreasing in a , 1 has no incentive to increase a . In particular, 1 has no incentive to increase a when $a = a^*$ and $b = b^*$.

The same argument also shows that $(x < a, y < b, 1 - y \leq a, 1 - x \leq b)$ cannot hold in equilibrium for any $a, b \in [0, 1] \times [0, 1]$, since 1 can increase her payoff by decreasing a . We will show later that $x \leq a$ or $y \leq b$ in Case pseudo-E cannot be slack in equilibrium, either.

Next, we will show that 1 has no incentive to decrease a when $a = a^*$ and $b = b^*$. We will also eliminate all other cases. Suppose that 1 deviates and commits to $a < a^*$. We will check the following possibilities.

Case 1: ($x \geq a$, $1 - y \leq a$, $y \leq b$, $1 - x \leq b$) holds in the bargaining subgame.

In this case, rewriting (1) and (2)

$$(1 - x) - \phi_2(b - (1 - x)) = \delta(y - \phi_2(b - y))$$

$$(1 - y) - \phi_1(a - (1 - y)) = \delta x$$

and solving for x and y , we obtain

$$y = \frac{(1 + \phi_1) - \delta x - \phi_1 a}{1 + \phi_1} \tag{9}$$

$$x = \frac{(1 + \phi_1)(1 + \phi_2)(1 - \delta) + \delta(1 + \phi_2)\phi_1 a - (1 - \delta)(1 + \phi_1)\phi_2 b}{(1 + \phi_2)(1 + \phi_1 - \delta^2)} \tag{10}$$

Now increase a . An increase in a increases x in (10) and decreases y in (9). Therefore, $y \leq b$ and $1 - x \leq b$ become slack. Also, $\frac{\partial(1-y)}{\partial a} = \frac{\phi_1}{1+\phi_1-\delta^2} < 1$, so that $1 - y \leq a$ becomes slack. If $x > a$ is slack initially, $x > a$ continues to hold if the increase in a is small enough. Then the conditions of Case 1 hold. This result and Lemma A1 imply that the new equilibrium is given by (9) and (10). Also, $V_1(a, b) = [p + (1 - p)\delta]x$ is increasing in a . Therefore, 1's optimal deviation that satisfies Case 1 also satisfies $x = a$, which implies $x \leq a$. That is, 1's optimal deviation that satisfies Case 1 reduces Case 1 to Case E. So, 1 cannot profitably deviate to an equilibrium which satisfies Case 1.

The same argument also proves that ($x > a$, $1 - y \leq a$, $y \leq b$, $1 - x \leq b$) cannot hold in any equilibrium for any $a, b \in [0, 1] \times [0, 1]$, since 1 can increase her payoff by increasing a .

Case pseudo-E again: Suppose that ($x = a$, $1 - y \leq a$, $y < b$, $1 - x \leq b$) holds in an equilibrium for some $a, b \in [0, 1] \times [0, 1]$. Then it implies ($x \geq a$, $1 - y \leq a$, $y \leq b$, $1 - x \leq b$),

which is Case 1, so the equilibrium is given by (9) and (10). Also, $x = a$ implies $1 - y < a$ and $y < b$ implies $1 - x < b$. Now consider a decrease in b . Since $1 - y \leq a$, $y \leq b$, $1 - x \leq b$ are all slack, they continue to hold if the decrease in b is small enough. Furthermore, x in (10) is decreasing in b . Therefore x increases in (10) so that $x \geq a$ continues to hold. These results and Lemma A1 imply that the new equilibrium is given by (9) and (10). It is easy to verify that $V_2(a, b) = [p\delta + (1 - p)][y - \phi_2(b - y)]$ is decreasing in b , so that $V_2(a, b)$ increases as b decreases. That is, if $(x = a, 1 - y \leq a, y < b, 1 - x \leq b)$ holds, then 2 can profitably deviate by decreasing b . This proves that $(x = a, 1 - y \leq a, y < b, 1 - x \leq b)$ cannot hold in any equilibrium for any $a, b \in [0, 1] \times [0, 1]$. Similarly, $(x < a, 1 - y \leq a, y = b, 1 - x \leq b)$ cannot hold in any equilibrium for any $a, b \in [0, 1] \times [0, 1]$.

Case 2: $(x \leq a, 1 - y \leq a, y \geq b, 1 - x \leq b)$ holds in the bargaining subgame.

In this case, rewriting (1) and (2)

$$(1 - x) - \phi_2(b - (1 - x)) = \delta y$$

$$(1 - y) - \phi_1(a - (1 - y)) = \delta(x - \phi_1(a - x))$$

and solving for x and y , we obtain

$$x = \frac{(1 + \phi_2) - \delta y - \phi_2 b}{1 + \phi_2} \tag{11}$$

$$y = \frac{(1 + \phi_1)(1 + \phi_2)(1 - \delta) + \delta(1 + \phi_1)\phi_2 b - (1 - \delta)(1 + \phi_2)\phi_1 a}{(1 + \phi_1)(1 + \phi_2 - \delta^2)} \tag{12}$$

Note that $x \leq a < a^*$ and $b = b^*$. Using (11) and (12), $x \leq a$ can be rewritten as

$$(1 + \phi_1)(1 + \phi_2) - (1 + \phi_1)\phi_2 b^* \leq (1 + \phi_2)(1 + \phi_1 + \delta)a$$

Substituting b^* , we obtain $a^* \leq a$, a contradiction. Therefore, given that 2 commits to b^* ,

by decreasing a , 1 cannot deviate to an equilibrium that satisfies Case 2.

Also, Case 2 is the symmetric of Case 1. If $(x \leq a, 1 - y \leq a, y > b, 1 - x \leq b)$, negotiator 2 can increase his payoff by increasing b . Therefore, $(x \leq a, 1 - y \leq a, y > b, 1 - x \leq b)$ cannot hold in any equilibrium for any $a, b \in [0, 1] \times [0, 1]$.

Case 3: $(x \geq a, 1 - y \leq a, y \geq b, 1 - x \leq b)$ holds in the bargaining subgame.

In this case, rewriting (1) and (2)

$$(1 - x) - \phi_2(b - (1 - x)) = \delta y$$

$$(1 - y) - \phi_1(a - (1 - y)) = \delta x$$

and solving for x and y , we obtain

$$y = \frac{(1 + \phi_1) - \delta x - \phi_1 a}{1 + \phi_1} \quad (13)$$

$$x = \frac{(1 + \phi_1)(1 + \phi_2 - \delta) + \delta \phi_1 a - (1 + \phi_1)\phi_2 b}{(1 + \phi_1)(1 + \phi_2) - \delta^2} \quad (14)$$

Now increase a . An increase in a increases x in (14) and decreases y in (13). Therefore, $1 - x \leq b$ becomes slack. Also, $\frac{\partial(1-y)}{\partial a} = \frac{\phi_1(1+\phi_1)}{(1+\phi_1)(1+\phi_2)-\delta^2} < 1$, so that $1 - y \leq a$ becomes slack. If $x \geq a$ and $y \geq b$ are slack initially, $x > a$ continues to hold if the increase in a is small enough. Then the conditions of Case 3 hold, so the new equilibrium is given by (13) and (14). Also, $V_1(a, b) = [p + (1 - p)\delta]x$ is increasing in a . Therefore, 1's payoff increases as a increases. If we continue to increase a , $x \geq a$ cannot become binding while $y \geq b$ is slack. Otherwise, we would arrive the contradiction of Case 2. Therefore, $y \geq b$ becomes binding first. By increasing a (and therefore increasing 1's payoff), we arrive $(x > a, 1 - y \leq a, y \leq b, 1 - x \leq b)$, which is Case 1. We can further increase 1's payoff by increasing a . Then, as we have seen in Case 1, the optimal deviation for 1 would satisfy Case E. That is, 1 cannot profitably deviate to an equilibrium in which Case 3 is satisfied.

The same argument also proves that $(x > a, 1 - y \leq a, y > b, 1 - x \leq b)$ cannot hold in any equilibrium for any $a, b \in [0, 1] \times [0, 1]$, since 1 can increase her payoff by increasing a .

Case 4: $(x \geq a, 1 - y \geq a, y \leq b, 1 - x \leq b)$ holds in the bargaining subgame.

In this case, rewriting (1) and (2)

$$(1 - x) - \phi_2(b - (1 - x)) = \delta(y - \phi_2(b - y))$$

$$1 - y = \delta x$$

and solving for x and y , we obtain

$$y = 1 - \delta x = \frac{(1 + \phi_2) + \delta \phi_2 b}{(1 + \phi_2)(1 + \delta)} \quad (15)$$

$$x = \frac{(1 + \phi_2) - \phi_2 b}{(1 + \phi_2)(1 + \delta)} \quad (16)$$

Also, $V_1(a, b) = [p + (1 - p)\delta]x$ as in Case E. Substituting $b = b^*$ in (16), we obtain $x < x^*$.

Therefore, if 1 deviates to an equilibrium that satisfies Case 4, then her payoff decreases.

Next, we will show that $(x > a, 1 - y \geq a, y \leq b, 1 - x \leq b)$ cannot hold in any equilibrium for any $a, b \in [0, 1] \times [0, 1]$. Suppose in contrary that it holds. Then x and y are given by (15) and (16). Note that $1 - y = \delta x < x$. Therefore, $1 - y \geq a$ implies $x > a$. Now start increasing a . Since a does not appear in (15) and (16), the increase in a will not change the solution until $1 - y \geq a$ becomes binding. When $1 - y \geq a$, we can equivalently write $1 - y \leq a$. Then we reduce Case 4 to Case 1. As we have shown in Case 1, a further increase in a increases 1's payoff. Therefore, $(x > a, 1 - y > a, y \leq b, 1 - x \leq b)$ cannot hold in any equilibrium.

Case 5: $(x \leq a, 1 - y \leq a, y \geq b, 1 - x \geq b)$ holds in the bargaining subgame.

In this case, rewriting (1) and (2)

$$(1 - x) - \phi_2(b - (1 - x)) = \delta(y - \phi_2(b - y))$$

$$1 - y = \delta x$$

and solving for x and y , we obtain

$$x = 1 - \delta y = \frac{(1 + \phi_1) + \delta\phi_1 a}{(1 + \phi_1)(1 + \delta)} \quad (17)$$

$$y = \frac{(1 + \phi_1) - \phi_1 a}{(1 + \phi_1)(1 + \delta)} \quad (18)$$

Note that $x \leq a < a^*$. Using (17), $x \leq a$ implies $a \geq \frac{1 + \phi_1}{1 + \delta + \phi_1} > a^*$, a contradiction. Therefore, given that 2 commits to b^* , by decreasing a , 1 cannot deviate to an equilibrium that satisfies Case 5.

Also, Case 5 is the symmetric of Case 4. If $(x \leq a, 1 - y \leq a, y > b, 1 - x > b)$, negotiator 2 can increase his payoff by increasing b . Therefore, $(x \leq a, 1 - y \leq a, y > b, 1 - x > b)$ cannot hold in any equilibrium for any $a, b \in [0, 1] \times [0, 1]$.

Case 6: $(x \geq a, 1 - y \geq a, y \geq b, 1 - x \leq b)$ holds in the bargaining subgame.

In this case, rewriting (1) and (2)

$$(1 - x) - \phi_2(b - (1 - x)) = \delta y$$

$$1 - y = \delta x$$

and solving for x and y , we obtain

$$y = \frac{(1 - \delta)(1 + \phi_1) + \delta\phi_2 b}{1 + \phi_2 - \delta^2} \quad (19)$$

$$x = \frac{(1 + \phi_1 - \delta) - \phi_2 b}{1 + \phi_2 - \delta^2} \quad (20)$$

Also, $V_1(a, b) = [p + (1 - p)\delta]x$ as in Case E. Note that $1 - y = \delta x < x$. Therefore, $1 - y \geq a$ implies $x > a$. Now start increasing a . Since a does not appear in (19) and (20), the increase in a will not change the solution until $1 - y \geq a$ becomes binding. When $1 - y \geq a$, we can equivalently write $1 - y \leq a$. Then we reduce Case 6 to Case 3. As we have shown in Case 3, a further increase in a increases 1's payoff. Also, by further increasing a (and therefore increasing 1's payoff), we arrive $(x > a, 1 - y \leq a, y \leq b, 1 - x \leq b)$, which is Case 1. We can further increase 1's payoff by increasing a . Then, as we have seen in Case 1, the optimal deviation for 1 would satisfy Case E. That is, 1 cannot profitably deviate to an equilibrium in which Case 6 is satisfied.

The same argument also proves that $(x > a, 1 - y > a, y > b, 1 - x \leq b)$ cannot hold in any equilibrium for any $a, b \in [0, 1] \times [0, 1]$, since 1 can increase her payoff by increasing a .

Case 7: $(x \geq a, 1 - y \leq a, y \geq b, 1 - x \geq b)$ holds in the bargaining subgame.

In this case, rewriting (1) and (2)

$$1 - x = \delta y$$

$$(1 - y) - \phi_1(a - (1 - y)) = \delta x$$

and solving for x and y , we obtain

$$x = \frac{(1 - \delta)(1 + \phi_2) + \delta\phi_1 a}{1 + \phi_1 - \delta^2} \quad (21)$$

$$y = \frac{(1 + \phi_2 - \delta) - \phi_1 a}{1 + \phi_1 - \delta^2} \quad (22)$$

Consider an increase in a . An increase in a increases x in (21) and decreases y in (22). Since $\frac{d(1-y)}{da} = \frac{\phi_1}{1+\phi_1-\delta^2} < 1$, $1 - y \leq a$ becomes slack. If $x \geq a$, $y \geq b$ and $1 - x \geq b$ continue to hold, the equilibrium x and y are given by (21) and (22). Therefore, $V_1(a, b) = [p + (1 - p)\delta]x$

increases. Continue to increase a . Check that $y \geq b$ cannot bind first. Otherwise, we obtain an equilibrium with $y \leq b$ and $1 - x \geq b$, which contradicts Lemma A4. $x \geq a$ cannot bind first either. Otherwise, $(x \leq a, 1 - y \leq a, y \geq b, 1 - x \geq b)$ holds, which is Case 5. Then we arrive the same contradiction as in Case 5. So, as we increase a , $1 - x \geq b$ binds first. Then $(x \geq a, 1 - y \leq a, y \geq b, 1 - x \leq b)$ holds, which is Case 3. As we have seen in Case 3, the optimal deviation for 1 would satisfy Case E. That is, 1 cannot profitably deviate to an equilibrium in which Case 7 is satisfied.

Also, Case 7 is the symmetric of Case 6. If $(x > a, 1 - y \leq a, y > b, 1 - x > b)$, negotiator 2 can increase his payoff by increasing b . Therefore, $(x > a, 1 - y \leq a, y > b, 1 - x > b)$ cannot hold in any equilibrium for any $a, b \in [0, 1] \times [0, 1]$.

Case 8: $(x \geq a, 1 - y \geq a, y \geq b, 1 - x \geq b)$ holds in the bargaining subgame.

In such an equilibrium, players face no cost on and off the equilibrium path. So, the outcome of such an equilibrium is the same as that of the standard Rubinstein bargaining game. That is, $x = y = \frac{1}{1+\delta}$. Starts increasing a . Since $1 - y = \frac{\delta}{1+\delta} < x$, when a reaches $\tilde{a} = \frac{\delta}{1+\delta}$, $1 - y \geq \tilde{a}$ binds. At this point, 1 continues to achieve the same payoff before the increase in a . Furthermore, we can rewrite this inequality as $1 - y \leq \tilde{a}$. We also have $x > \tilde{a}$, $y > b, 1 - x > b$. Therefore, we can equally solve for the equilibrium with $(x > a, 1 - y \leq a, y > b, 1 - x \leq b)$, which is Case 3. As we have seen in Case 3, the optimal deviation for 1 would satisfy Case E. That is, 1 cannot profitably deviate to an equilibrium in which Case 8 is satisfied.

The same argument also proves that $(x > a, 1 - y > a, y > b, 1 - x > b)$ cannot hold in any equilibrium since 1 can increase her payoff by increasing a .

Therefore, the described strategy profile is the unique equilibrium. This completes the

proof. Q.E.D.

3 Citizens Imposing Audience Costs

In our conclusion, we argue that our findings provide a microfoundation for audience costs in a repeated negotiations framework. We now provide a formal proof to this claim.

Consider the following extension of our model. There are two publics, P_1 and P_2 . P_1 and P_2 each contains a continuum of agents. We will consider symmetric strategies for the members of each public. That is, if two members of P_i are symmetric, they adopt the same strategy. Therefore, we treat each P_i as a single agent.

The two publics play the following stage game repeatedly at $t = 1, 2, \dots$

- A negotiator $N_i^t \in P_i$ is elected randomly in the beginning of a period.
- Then N_1^t and N_2^t play the game described in the main text. That is, they simultaneously commit to a_{ti} ; then they play the Rubinstein bargaining game. Let p_i denote the probability of N_i^t making the first offer; $p_1 + p_2 = 1$.
- Let π_{ti} be the share that N_i^t secures in the bargaining. At the end of t , each P_i simultaneously decides whether to impose the audience cost $C_i(\pi_{ti}, a_{ti})$. If $C_i(\pi_{ti}, a_{ti}) > 0$, i.e. if N_i^t backs down from his public commitment, then P_i 's cost of imposing $C_i(\pi_{ti}, a_{ti})$ is ψ_i .
- The payoff of N_i^t at the end of t is given by $\pi_{ti} - C_i(\pi_{ti}, a_{ti})$ if P_i imposes the audience cost; it is given by π_{ti} if P_i does not impose the cost.
- Each player discounts future payoffs by β . Now we interpret $1 - \delta$ as the probability that a bargaining breaks down after an offer is rejected. If a bargaining breaks down,

it yields a share of zero to each side that period. All of our previous analysis is valid under this alternative interpretation of δ .

Proposition E1: If $\phi_i > 0$ and

$$\psi_i < \frac{\beta\phi_i}{1 - \beta(1 - p_j)} \frac{p_i(\delta + \phi_j) + p_j(1 + \phi_j)}{(1 + \delta + \phi_i + \phi_j)(1 + \delta + \phi_j)} \quad (23)$$

for each $i = 1, 2$ and $j \neq i$, each P_i imposes the audience cost in every period in a subgame perfect Nash equilibrium of the repeated game.

Proof:

Let $\hat{\phi} = (\hat{\phi}_1, \hat{\phi}_2)$ denote a state. Consider the following three states: $\hat{\phi}^{12} = (\phi_1, \phi_2)$, $\hat{\phi}^1 = (\phi_1, 0)$, $\hat{\phi}^2 = (0, \phi_2)$. We interpret $\hat{\phi}_t$ as the cost coefficients of the stage game that is being played by N_1^t and N_2^t in period t . For example, if both N_1^t and N_2^t believe that their publics will impose the audience cost if they back down from their public commitment in t , then N_1^t and N_2^t plays the stage game associated with $\hat{\phi}_t = \hat{\phi}^{12} = (\phi_1, \phi_2)$. On the other hand, if N_1^t and N_2^t believe that only N_1^t faces the audience cost, then they play the stage game associated with $\hat{\phi}_t = \hat{\phi}^1 = (\phi_1, 0)$.

The following strategy profile is a subgame perfect Nash equilibrium of the repeated game and it yields the outcome in the proposition:

- The state starts with $\hat{\phi}_1 = \hat{\phi}^{12} = (\phi_1, \phi_2)$.
- In period $t \geq 1$, if the state is $\hat{\phi}^{12}$ or $\hat{\phi}^i$, then P_i imposes the audience cost when N_i^t backs down from his public commitment. Otherwise, P_i does not impose the audience cost.

- If $\hat{\phi}_t = \hat{\phi}^{12}$ or $\hat{\phi}_t = \hat{\phi}^i$ and P_i fails to impose the audience cost, then the state switches to $\hat{\phi}^j$.
- N_i^t plays his subgame perfect equilibrium strategy of the stage game with cost coefficients $\hat{\phi}_t$. If $\hat{\phi}_t = \hat{\phi}^{12}$ or $\hat{\phi}_t = \hat{\phi}^i$, then N_i^t 's equilibrium strategy is unique. If $\hat{\phi}_t = \hat{\phi}^j$, then N_i^t commits to zero. (In fact, in that case any public commitment by N_i^t can be an equilibrium strategy, while N_j^t has a unique equilibrium strategy. However, all the equilibria yield the same outcome)

Given P_1 and P_2 's strategies prescribed by this strategy profile, N_1^t and N_2^t 's strategies constitute a subgame perfect equilibrium of the game that is induced by P_1 and P_2 's strategies. This result follows from our main result just by noting that P_i 's prescribed strategy to impose the audience cost depends only on P_1 and P_2 past behavior, not on the behavior of N_1^s or N_2^s , $s \leq t$.

Next we show that P_1 and P_2 's strategies constitute an equilibrium.

Let V^τ denote the expected payoff of $\tau \in \{P_1, P_2\}$.

Suppose that the state is $\hat{\phi}^{12}$. If the players play the above strategy profile, P_1 and P_2 impose audience costs every following period. Therefore, the state remains to be $\hat{\phi}^{12}$ and negotiators repeatedly play the stage game with cost coefficients $\hat{\phi}^{12}$. Using our main result, each N_i^t commits to $a_{ti}^{12} = \frac{1+\phi_i}{1+\delta+\phi_i+\phi_j}$; in the following bargaining game N_i^t offers to take a share of a_{ti}^{12} with probability p_i and takes it, in which case N_j^t suffers an audience cost of $\phi_j \frac{1-\delta}{1+\delta+\phi_i+\phi_j}$ and P_j suffers ψ_j , the cost of imposing the audience cost. For $i = 1, 2$ and $j \neq i$, we can compute P_i 's continuation payoff as follows:

$$V^{P_i}(\hat{\phi}^{12}) = \frac{1}{1-\beta} \left[p_i \frac{1+\phi_i}{1+\delta+\phi_i+\phi_j} + p_j \left(\frac{\delta+\phi_i}{1+\delta+\phi_i+\phi_j} - \psi_i \right) \right]$$

Suppose that the state is $\hat{\phi}^i$. If the players play the above strategy profile, P_i imposes the audience cost every following period and P_j does not impose any cost. Therefore, the state remains to be $\hat{\phi}^i$ and negotiators repeatedly play the stage game with cost coefficients $\hat{\phi}^i$. Using our main result, N_i^t commits to $a_{ti}^i = \frac{1+\phi_i}{1+\delta+\phi_i}$; N_j^t commits to zero; in the following bargaining game N_i^t offers to take a share of $\frac{1+\phi_i}{1+\delta+\phi_i}$ with probability p_i and takes it; N_j^t offers to take a share of $\frac{\delta+\phi_i}{1+\delta+\phi_i}$ with probability p_j and takes it, in which case N_i^t suffers an audience cost of $\phi_i \frac{1-\delta}{1+\delta+\phi_i}$ and P_i suffers ψ_i , the cost of imposing the audience cost. We can compute players' continuation payoffs as follows:

$$V^{P_i}(\hat{\phi}^i) = \frac{1}{1-\beta} \left[p_i \frac{1+\phi_i}{1+\delta+\phi_i} + p_j \left(\frac{\delta+\phi_i}{1+\delta+\phi_i} - \psi_i \right) \right]$$

$$V^{P_j}(\hat{\phi}^i) = \frac{1}{1-\beta} \left[p_i \frac{\delta}{1+\delta+\phi_i} + p_j \frac{1}{1+\delta+\phi_i} \right]$$

Similarly, we can compute the continuation payoffs in state $\hat{\phi}^j$ as follows:

$$V^{P_i}(\hat{\phi}^j) = \frac{1}{1-\beta} \left[p_i \frac{1}{1+\delta+\phi_j} + p_j \frac{\delta}{1+\delta+\phi_j} \right]$$

$$V^{P_j}(\hat{\phi}^j) = \frac{1}{1-\beta} \left[p_i \left(\frac{\delta+\phi_j}{1+\delta+\phi_j} - \psi_j \right) + p_j \frac{1+\phi_j}{1+\delta+\phi_j} \right]$$

Suppose that the state is $\hat{\phi}^{12}$. Given that other players play their strategies prescribed by the above profile, if P_i plays its prescribed strategy, then its continuation payoff is $V^{P_i}(\hat{\phi}^{12})$. Suppose that P_i deviated from this strategy for one period and does not impose the audience cost when N_j^t makes an offer and N_i^t backs down from his public commitment by accepting N_j^t 's offer. Then the state switches to $\hat{\phi}^j$ forever. So, P_i 's payoff from this one period deviation can be computed as

$$\frac{\delta+\phi_i}{1+\delta+\phi_i+\phi_j} + \beta V^{P_i}(\hat{\phi}^j)$$

P_i does not deviate if

$$\frac{\delta + \phi_i}{1 + \delta + \phi_i + \phi_j} - \psi_i + \beta V^{P_i}(\hat{\phi}^{12}) > \frac{\delta + \phi_i}{1 + \delta + \phi_i + \phi_j} + \beta V^{P_i}(\hat{\phi}^j)$$

equivalently

$$\psi_i < \beta \left[V^{P_i}(\hat{\phi}^{12}) - V^{P_i}(\hat{\phi}^j) \right] \quad (24)$$

that is

$$\psi_i < \frac{\beta \phi_i}{1 - \beta(1 - p_j)} \frac{p_i(\delta + \phi_j) + p_j(1 + \phi_j)}{(1 + \delta + \phi_i + \phi_j)(1 + \delta + \phi_j)}$$

Suppose that the state is $\hat{\phi}^i$. Given that other players play their strategies prescribed by the above profile, if P_i plays its prescribed strategy, then its continuation payoff is $V^{P_i}(\hat{\phi}^i)$. Suppose that P_i deviated from this strategy for one period and does not impose the audience cost when N_j^t makes an offer and N_i^t backs down from his public commitment by accepting N_j^t 's offer. Then the state switches to $\hat{\phi}^j$ forever. So, P_i 's payoff from this one period deviation can be computed as

$$\frac{\delta + \phi_i}{1 + \delta + \phi_i} + \beta V^{P_i}(\hat{\phi}^j)$$

P_i does not deviate if

$$\frac{\delta + \phi_i}{1 + \delta + \phi_i} - \psi_i + \beta V^{P_i}(\hat{\phi}^j) > \frac{\delta + \phi_i}{1 + \delta + \phi_i} + \beta V^{P_i}(\hat{\phi}^j)$$

equivalently

$$\psi_i < \beta \left[V^{P_i}(\hat{\phi}^i) - V^{P_i}(\hat{\phi}^j) \right] \quad (25)$$

It is easily verified that $V^{P_i}(\hat{\phi}^{12}) < V^{P_i}(\hat{\phi}^i)$. So 24 implies 25 and P_i does not deviate in state $\hat{\phi}^i$, if 24 holds.

If the state is $\hat{\phi}^j$, then P_i does not have an opportunity to deviate, because N_i^t commits to zero, which is less than N_j^t 's offer.

Therefore, if 24 is satisfied for $i = 1, 2$, then, the above strategy profile constitutes a subgame perfect equilibrium of the repeated game. Furthermore, in equilibrium, the state remains to be $\hat{\phi}^{12}$ and each P_i always imposes audience cost when N_i^t backs down from his public commitment. This completes the proof. Q.E.D.

Although Proposition E1 is a sufficient proof of our argument, it does not rule out the possibility of an equilibrium in which P_1 and P_2 cooperate in not imposing the audience costs. Next we show that P_1 and P_2 may fail to cooperate in avoiding the cost of imposing audience costs.

Proposition E2: There exists a set of parameters $\{(p_i, \phi_i, \psi_i)_{i=1,2}, \delta, \beta\}$ such that there exists no Nash equilibrium in which P_1 and P_2 do not impose audience costs.

Proof:

Let $\hat{\phi}^0 = (0, 0)$. If both P_1 and P_2 can commit to not imposing the audience costs, then the stage game played among negotiators turns to be the standard Rubinstein bargaining game in which the proposer gets $\frac{1}{1+\delta}$. Then P_i 's payoff can be computed as

$$V^{P_i}(\hat{\phi}^0) = \frac{1}{1-\beta} \left[p_i \frac{1}{1+\delta} + p_j \frac{\delta}{1+\delta} \right]$$

Next we will find conditions under which committing to not imposing the audience costs cannot be an equilibrium.

Consider an equilibrium in which P_1 and P_2 never impose audience costs on the equilibrium path. Suppose that P_i deviates for one period by imposing the audience cost on its negotiator. Given that P_j is not imposing the audience costs on the equilibrium path, P_i 's payoff from its deviation is given by

$$(1-\beta)V^{P_i}(\hat{\phi}^i) + \beta(P_i\text{'s continuation payoff})$$

P_i does not make this deviation if

$$V^{P_i}(\hat{\phi}^0) > (1 - \beta)V^{P_i}(\hat{\phi}^i) + \beta(P_i\text{'s continuation payoff})$$

Since we are trying to find the conditions under which this inequality fails to hold - so that not imposing audience costs cannot be an equilibrium outcome- we will use P_i 's minmax payoff on the right hand side of this inequality and look for conditions that satisfy the following

$$V^{P_i}(\hat{\phi}^0) < (1 - \beta)V^{P_i}(\hat{\phi}^i) + \beta V_{\min\max}^{P_i} \quad (26)$$

P_i 's minmax payoff is the minimum payoff that P_j can impose on P_i by committing to a certain strategy. Since P_j has only two actions to choose from in the stage game, either imposing the audience cost or not imposing it, P_j can minmax P_i either by committing to imposing the audience cost forever or not imposing it forever. In each case, P_i can optimally choose whether to impose its audience cost or not. So $V_{\min\max}^{P_i}$ can be computed as follows:

$$V_{\min\max}^{P_i} = \min \left\{ \max \left\{ V^{P_i}(\hat{\phi}^{12}), V^{P_i}(\hat{\phi}^j) \right\}, \max \left\{ V^{P_i}(\hat{\phi}^i), V^{P_i}(\hat{\phi}^0) \right\} \right\}$$

Note that $V^{P_i}(\hat{\phi}^{12}) \leq \max \left\{ V^{P_i}(\hat{\phi}^i), V^{P_i}(\hat{\phi}^0) \right\}$ since $V^{P_i}(\hat{\phi}^{12}) < V^{P_i}(\hat{\phi}^i)$. If also $V^{P_i}(\hat{\phi}^j) \leq V^{P_i}(\hat{\phi}^{12})$, then $V_{\min\max}^{P_i} = V^{P_i}(\hat{\phi}^{12})$. So suppose that $V^{P_i}(\hat{\phi}^j) \leq V^{P_i}(\hat{\phi}^{12})$, that is

$$\psi_i \leq \frac{\phi_i}{p_j} \frac{p_i(\delta + \phi_j) + p_j(1 + \phi_j)}{(1 + \delta + \phi_i + \phi_j)(1 + \delta + \phi_j)} \quad (27)$$

Then 26 holds if

$$V^{P_i}(\hat{\phi}^0) < (1 - \beta)V^{P_i}(\hat{\phi}^i) + \beta V^{P_i}(\hat{\phi}^{12})$$

equivalently

$$\psi_i < \frac{\phi_i}{p_j} \frac{p_i\delta + p_j}{(1 + \delta)(1 + \delta + \phi_i)} - \beta \frac{\phi_j}{p_j} \frac{p_i(1 + \phi_i) + p_j(\delta + \phi_i)}{(1 + \delta + \phi_i)(1 + \delta + \phi_i + \phi_j)} \quad (28)$$

The right hand side of 28 has to be positive for the existence of ψ_i that satisfies 28. Now substitute $\beta = 1$ and check that

$$\frac{\phi_i}{p_j} \frac{p_i \delta + p_j}{(1 + \delta)(1 + \delta + \phi_i)} > \frac{\phi_j}{p_j} \frac{p_i(1 + \phi_i) + p_j(\delta + \phi_i)}{(1 + \delta + \phi_i)(1 + \delta + \phi_i + \phi_j)}$$

is equivalent to

$$\phi_j < \frac{\phi_i(p_i \delta + p_j)(1 + \delta + \phi_i)}{(1 + \delta)(p_i \delta + p_j) + \phi_i(1 - p_j)(1 - \delta)}$$

so that if ϕ_j is small enough, there exists a set of parameters that satisfies 27 and 28. In this case, in every Nash equilibrium of the game, P_i imposes its audience costs. This completes the proof. Q.E.D.