

An evolutionary theory of inflation inertia*

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Abstract

We provide a simple theory of inflation inertia in a staggered price setting framework a la Calvo (1983). Contrary to Calvo's formulation, the frequency of price changes is allowed to vary according to an evolutionary criterion. Inertia is the direct result of gradual adjustment in this frequency following a permanent change in the rate of money growth.

1 Introduction

The recent literature on monetary policy has widely used the sticky price model proposed by Calvo (1983) as a simple way of generating the so-called (New Keynesian) Phillips curve, a negative relation between inflation and the output gap – see Woodford (1994) or Clarida, Gali and Gertler (1999), amongst others. However, as pointed out by Fuhrer and Moore (1995), the Calvo

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model does not succeed in reproducing the observed persistence of inflation. In Calvo (1983), even if only a few firms change prices at any period, the assumption of rational expectations implies that these firms fully understand the price effects of changes in the environment. This in turn has important implications for the behavior of inflation. In particular, when the economy faces an unexpected change in the rate of money growth, the forward-looking behavior of those few firms currently changing prices is enough for the economy to anticipate the effect of the shock on inflation. In other words, firms currently setting prices will choose their prices in such a way that the overall level of inflation will immediately jump to the new steady state.

In this paper, we use an evolutionary game approach to solve a Calvo (1983) economy with a cash-in-advance constraint as a simple way of generating inflation inertia. The description of the economy is taken directly from Calvo (1983), but money is introduced following Clower (1967) instead of Sidrauski (1967). When a rational expectations equilibrium is computed for this economy, the standard result that inflation automatically adjusts to a permanent, unexpected shock in the growth rate of money supply holds. However, when firms play an evolutionary game in prices, instead of behaving consistently with rational expectations, inflation responds slowly and gradually to exogenous shocks to the money supply growth rate. In the evolutionary economy, a firm receiving a price-change signal follows a simple rule of thumb relating its price to the observed money supply. The probability of observing a price-change signal, and thus the frequency with which firms change prices, evolves depending on the performance of this price rule relative to any other price observed in the economy. When firms changing prices perform better (worse) than those that keep posting their old prices, price change signals become more (less) frequent. We refer to this dynamic adjustment as the Darwinian dynamics, to highlight the underlying idea that successful behavior tends to spread in the population. Within this setup, the response of inflation to a permanent negative shock on the rate of money growth is gradual. Even though price-changing firms keep playing the same price rule as before, inflation will tend towards the new rate of money growth due to the Darwinian dynamics. That is the

dynamic adjustment of inflation will work through the adjustment in the probability of observing a price-change signal. That adjustment will be slow and gradual, delivering in a simple, intuitive way inflation inertia.

Our approach is not unique in delivering the result of inflation inertia. A growing recent literature seeks ways of obtaining inflation persistence by modifying the sticky price model that originates in Calvo's staggered price setting formulation. The most notable of such attempts is due to Mankiw and Reis (2002). They stress the role of sticky information as an alternative to sticky prices. Under sticky information, firms revise prices every period but their decisions are not always based on current information. Inertia results from the fact that some price setters decide on price changes based on past information. A similar reasoning explains why a sticky price framework augmented with price indexation delivers the same result.¹ In an evolutionary framework, information spreads across the economy through evolution, i.e. the replication of high performing behavior. In this paper, changes in the environment affect price-pioneers' profits making its size in population adjust. Inertia results because news are not observed and, more important, agents don't adapt their behavior to them, but the economy learns slowly through the replication of those behaviors that are most adapted to the new environment.

2 The Economy

The economy in this paper is very close to the model of Calvo (1983), with the difference that money is introduced following Clower (1967) instead of Sidrauski (1967). The fundamentals are described as in the general equilibrium tradition. However, instead of using an equilibrium concept to formalize the behavior of agents and markets, an evolutionary game in prices is assumed and its outcome is computed.

¹For a detailed analysis of the similarities and differences between the two models see Trabandt (2006). Yet another interesting recent development is due to Mackowiak and Wiederholt (2006) who derive inflation and price setting implications of rational inattention, an idea originally suggested by Chris Sims (2003).

Time is continuous. There is a sole, perishable and divisible good. The economy is populated by a continuum of firms in the interval $[0, 1]$. At every instant, each firm receives, as manna from heaven, a strictly positive endowment of size q . Therefore, the total amount of goods available at t is q . There is a continuum of individuals; the representative individual has infinite life, time additive preferences with constant, strictly positive discount rate, owns firms and holds money. Money M_t is issued by a central bank and is permanently increasing at the instantaneous rate $\mu > 0$

$$M_t = M_0 e^{\mu t}.$$

Any increase in money supply is distributed across individuals as a lump-sum transfer.

In the spirit of Clower (1967), money is required for transactions – any form of barter is forbidden – meaning that money plays the role of both a *unit of account* and a *medium of exchange*. Individuals hold money and use their money holdings to buy goods. Firms collect profits in the form of money and give the money back to individuals as dividends.

What can we learn from equilibrium theory? Under standard conditions on preferences, an equilibrium path with binding cash-in-advance constraint exists. Such an equilibrium is consistent with the *quantity theory of money* $P_t q = M_t$, where P_t is the *equilibrium price* of the physical good and the velocity of money is one.² In the evolutionary economy described below the equilibrium price plays no role, since agents do not use it to take any economic decision. However, we will use the equilibrium outcome as a benchmark to analyze the performance of the evolutionary economy, by comparing evolutionary prices to the equilibrium price.

The evolutionary game we propose is extremely simple and very close to the price setting process in Calvo (1983). Firms keep announcing the same price until they receive a price-change signal. Firms receiving such a signal, instead of computing an optimal price consistent with rational expectations as in Calvo, set a price proportional to the observed money supply. This simple rule of thumb allows prices to follow money. We call these firms price-pioneers; they

²See Lucas and Stokey (1987) and Woodford (1994).

understand that prices must increase since money is growing and they implement it in a simple way. They play a similar role as Calvo players in the Calvo model. The price-change signal arrives with an instantaneous rate that adjusts upwards or downwards depending on whether the price-pioneer is making the largest profits. This is a simple application of the Darwinian principle that successful behavior tends to replicate. Let us now be more precise on the definition of the evolutionary game.

At any time t , firms announce prices and engage on satisfying demand at these prices until they run out of stock. As a consequence, it may be that some firms keep unsold units of the perishable good, which cannot be transferred to the next period. In such a case, these firms are said to be quantity constrained by a demand shortage. The representative individual uses her money holdings to buy as much of the physical good as she can. She is assumed to observe all current prices, order firms by prices and buy following the order of prices, the lowest price first. As a result, either individuals spend the total available amount of money M_t or all firms run out of stock. In order to ensure individuals are identical, we make the assumption that any excess demand is proportionally distributed across individuals. We say that the representative individual is quantity constrained when she cannot buy as many units as she would like, because of a shortage of supply.³

Firms' prices are set according to the following rules. At the initial time $t = 0$, firms carry prices from the past. The relevant information on the initial distribution of prices can be summarized by \mathcal{P}_0 , the average price announced at $t = 0$. At any time $t \geq 0$, firms keep announcing their previous prices until they receive a price-change signal, in which case they set a price proportional to the money supply, i.e., $\hat{\eta}M_t$, $\hat{\eta} > 0$. We refer to the firms that receive a price change signal as *price-pioneers*. In order to follow this price rule, the only information requirement is that price-pioneers observe the money supply M_t . Note that this rule is equivalent to setting a

³A large literature in the seventies and eighties has developed equilibrium concepts to deal with situations where markets are not cleared by prices. See Benassy (1982) and Dreze (1974).

price proportional to the equilibrium price P_t , which was defined previously as $P_t = M_t/q$. That is, one can equivalently express the pioneers' price setting rule as $(1 + \eta) P_t$, with $\eta = \hat{\eta}q - 1$. We use this notation for convenience from now on, but it is important to clarify that we do not require price pioneers to observe the equilibrium price P_t ; observing M_t is enough. Since the pioneer always sets the largest price let us assume that $\eta > 0$, otherwise the average announced price would be smaller than the equilibrium price and the economy would (by construction) permanently be in a situation of excess demand.

The probability density of receiving a price-change signal during a period of length k from time t is assumed to be

$$v_{t+k} e^{-\int_0^k v_{t+z} dz},$$

with $v_t > 0$ representing the density of price-pioneers in the distribution of firms at time t . This is similar to Calvo (1983), the difference amounting to the fact that in Calvo v_t is constant.

At any time t , let us denote by p_{kt} the price set by price-pioneers at $t - k$, $k \geq 0$. By definition of the pioneer's price rule, $p_{kt} = (1 + \eta) P_{t-k} = (1 + \eta) e^{-\mu k} P_t$, since the equilibrium price follows money by definition. Let h_{kt} be the density of the $t - k$ pioneers still announcing p_{kt} at time t , which is $h_{kt} = v_{t-k} e^{-\int_0^k v_{t-z} dz}$. Lastly, the average price announced by firms at time $t \geq 0$, which we denote by \mathcal{P}_t , can be expressed as

$$\mathcal{P}_t = \mathcal{P}_0 e^{-\int_0^t v_z dz} + \int_k p_{kt} h_{kt} dk = \mathcal{P}_0 e^{-\int_0^t v_s ds} + (1 + \eta) \mathcal{A}_t P_t, \quad (1)$$

where

$$\mathcal{A}_t = \int_0^t e^{-\mu(t-z)} v_z e^{-\int_z^t v_s ds} dz.$$

We have used the variable change $z = t - k$ to obtain \mathcal{A}_t . When t goes to infinity, initial conditions vanish and the ratio of the average to the equilibrium price converges to

$$\frac{\mathcal{P}_t}{P_t} = (1 + \eta) \frac{v}{v + \mu}$$

if v_t converges to a constant value v .

From the description of the evolutionary game above, the economy is in excess supply or excess demand depending on

$$M_t \leq q\mathcal{P}_t \quad \text{or equivalently} \quad \frac{P_t}{\mathcal{P}_t} \leq 1.$$

The second relation follows directly from the first after substituting the definition of the equilibrium price $P_t = M_t/q$. The following Lemma shows that there is a one-to-one relationship between excess supply and excess demand on the one hand and the pioneers' performance on the other hand.

Lemma 1 *If $\mathcal{P}_t \leq P_t$ then the pioneer is making the largest profits, otherwise the pioneer is not making the largest profits.*

Proof. Let x_{kt} and Π_{kt} denote goods sold and profits made by a firm that was a price pioneer at $t - k$ and has not changed its prices since. Thus for a current price-pioneer, goods sold are denoted by x_{0t} and profits by Π_{0t} . When $\mathcal{P}_t \leq P_t$, the economy is in excess demand, implying that $x_{0t} = q$ and $\Pi_{0t} \geq \Pi_{kt}$ for all $k > 0$. When $\mathcal{P}_t > P_t$, the economy is in excess supply. Since pioneers are setting the largest price and have zero measure in the distribution of prices, $x_{0t} = 0$ and $\Pi_{0t} = 0$, implying that price-pioneers are not making the largest profits. That is because prices are strictly positive and some firms are selling q , therefore making strictly positive profits.

■

In the first situation, the average price is lower than the equilibrium price, implying that the economy is in excess demand, i.e. consumers cannot spend all their money holdings. In this case, the pioneer is setting the largest price, running out of stock, and making the largest profits. In the other case, the economy is facing an excess supply. Since pioneers are charging the largest price, they receive no demand and make zero profits.

Lastly, let us specify the Darwinian dynamics as

$$\dot{v}_t = \lambda \frac{P_t - \mathcal{P}_t}{\mathcal{P}_t} \tag{2}$$

where $\lambda > 0$ is the velocity of the evolutionary process. From the previous Lemma, $P_t \geq \mathcal{P}_t$ is an indicator of the price-pioneers' performance. The Darwinian dynamics assumes that the probability density of being a pioneer increases (decreases) when pioneers are (not) making the largest profits. This is an application of the Darwinian principle saying that successful behavior tends to replicate in the population. Note that the absolute size of both the equilibrium and the evolutionary price grows with money holdings. The observed absolute distances between the two will therefore also increase with time. To ensure the evolutionary adjustment is not affected by this artificial increase, we divide the right-hand-side by \mathcal{P}_t , i.e. we relate the rate of change in v to *relative* distance from the equilibrium price.

3 Inflation inertia

In this section, we study the dynamics of the evolutionary economy and compare them to the dynamics of the Calvo model. The main objective is the analysis of inflation inertia in the case of a permanent change in the rate of money growth. We show that, contrary to the behavior of the Calvo model where inflation jumps instantaneously to the new rate of money growth, in the evolutionary economy inflation moves gradually from its past level to the new one. That is, there is no inflation inertia in the Calvo model, but the evolutionary economy generates it since there is high persistence in past behavior.

3.1 Evolutionary Economy

By differentiating (1) with respect to t , we get

$$\dot{\mathcal{P}}_t = v_t ((1 + \eta) P_t - \mathcal{P}_t). \quad (3)$$

Note that only pioneers change prices and they do so with probability density v_t . On average, they move from the average announced price \mathcal{P}_t to the new price $(1 + \eta) P_t$.

It is easy to see that (3) is a particular version of the Phillips curve. Let us define the output gap, a measure of excess supply, as

$$y_t \equiv \frac{q - \frac{M_t}{\mathcal{P}_t}}{q}$$

where M_t/\mathcal{P}_t is the amount of goods individuals would like to buy at the announced prices and q is the total endowment, a measure of capacity. Using this definition, we can write (3) as

$$\pi_t \equiv \frac{\dot{\mathcal{P}}_t}{\mathcal{P}_t} = v_t (\eta - (1 + \eta) y_t),$$

which implies that inflation π_t depends negatively on the output gap.

Let us now rewrite the dynamic system. From the definition of the equilibrium price $P_t = \mathcal{P}_t$, the output gap can be written as $y_t = 1 - p_t$, with $p_t = P_t/\mathcal{P}_t$. After straightforward algebra (3) becomes

$$\frac{\dot{p}_t}{p_t} = \mu - v_t ((1 + \eta) p_t - 1). \quad (4)$$

From the definition of p_t , the Darwinian dynamics (2) can be written as

$$\dot{v}_t = \lambda (p_t - 1). \quad (5)$$

The economy has now been reduced to equations (4) and (5), an ODE system in p and v . Since the system is purely backward looking, both p and v are state variables, implying that initial conditions p_0 and v_0 need to be specified. From (4) and (5), $p_t = 1$ and $v_t = \mu/\eta$ at steady state. At the steady state of the evolutionary economy, the average price announced by firms is equal to the equilibrium price, implying that the economy is in equilibrium. The instantaneous probability of being a pioneer depends positively on the growth rate of money supply and negatively on η , the constant in the pioneer's price setting rule. When the money supply is growing at a large rate, nominal aggregate demand is growing at a large rate too. In a sticky price framework, pioneers are making large profits since other firms are not changing their prices. Evolution increases the number of pioneers, which increases inflation and reduces pioneer's profits up to the point where the evolutionary economy is at steady state. This is very

different from the Calvo economy, where the probability of receiving a price-change signal is constant and the economy adjusts through changes in Calvo players' prices. In the evolutionary economy, the probability of receiving a price-change signal is endogenous and positively related to the growth rate of money, which implies the desirable prediction that in a high inflation environment firms change prices more frequently.⁴

The eigenvalues of the linearized system (4)-(5) are

$$-\frac{(1+\eta)\mu}{\eta} \pm \sqrt{\left(\frac{(1+\eta)\mu}{\eta}\right)^2 - 4\lambda\eta}.$$

Both eigenvalues are negative real numbers if

$$\lambda \leq \left(\frac{(1+\eta)\mu}{\eta}\right)^2 \frac{1}{4\eta},$$

otherwise they are complex numbers with negative real part. The solution does converge, but it may converge by oscillations if the Darwinian dynamics adjust at high speed.

We study inflation inertia by analyzing the reaction of (4)-(5) to a permanent, negative shock to the rate of money growth, under the assumption that the economy was initially at steady state. The unit of time is a quarter. The quarterly rate of money growth was 2.5% before the shock and the new one is 2%, a 2 percentage points reduction in the annual growth rate. The instantaneous rate of price-change signals decreases from 0.30 to 0.24 in the new steady state. As it can be observed in Figure 1 inflation is persistent, with an impulse response function close to the one generated by sticky information.⁵ Pioneers keep playing the same price rule as before the shock and inflation reduces gradually at the speed with which the Darwinian dynamics reduces the size of price-pioneers in the total population.⁶

⁴A similar dynamic system emerges from an economy with constant probability density v and a simple learning process in η of the type $\dot{\eta}_t = \lambda(p_t - 1)$, with $\lambda > 0$, where the pioneers rule adjusts upwards or downwards depending on pioneers' profits. For example, under excess demand $p > 1$, pioneers can increase profits by increasing η .

⁵See Mankiw and Reis (2002), the bottom picture of Figure II on page 1305.

⁶Inertia depends crucially on the velocity of the replicator dynamics process. We set $\lambda = 0.46$, implying that

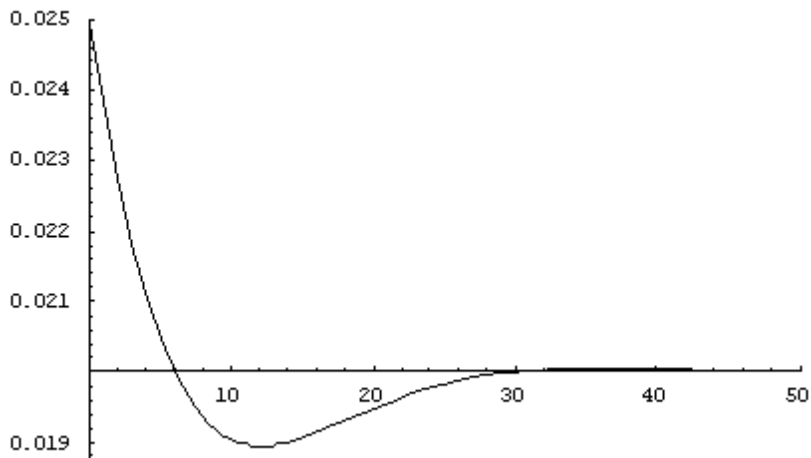


Figure 1: Reaction of quarterly inflation to a permanent reduction in the quarterly growth rate of money from 0.025 to 0.02

3.2 Calvo (1983)

To understand better the novelty of our result, we compare it to the cash-in-advance version of Calvo (1983). Equilibrium in the Calvo model is described by an ODE system for inflation π_t and real money balances m_t :⁷

$$\frac{\dot{m}_t}{m_t} = \mu - \pi_t \quad (6)$$

$$\dot{\pi}_t = b(q - m_t). \quad (7)$$

Since nominal money balances are a stock and prices are sticky, real money balances are a predetermined variable, with given initial condition $m_0 > 0$. Inflation is not predetermined, even if the average price level is. Equation (6) comes directly from the definition of real balances, which stop moving when inflation is equal to the growth rate of money. Equation (7) results from eigenvalues are complex. The eigenvalues are real for $\lambda < 0.2$, in which case inflation converges monotonically in less than 20 quarters. For very large values of λ , inflation converges by oscillations moving at a very high frequency.

⁷See equations (20b,c) in Calvo (1983), with $c = m$ because of the cash-in-advance constraint.

the price setting process and says that inflation adjusts to the output gap, i.e., the difference between aggregate capacity q and aggregate demand m . Parameter b represents the speed of inflation adjustment, which depends negatively on the average time before a price change occurs (the inverse of the rate of price-change signals v). Remember that in Calvo the average price is a state variable. When the economy is in excess supply, prices have to increase; since they cannot jump instantaneously, inflation has to increase to reduce the output gap. At steady state, inflation is equal to the rate of money growth ($\pi_t = \mu$) and the endowment is fully consumed ($m_t = q$). The system (6)-(7) is saddle path stable, with both m and π increasing monotonically if the economy is initially in excess supply, i.e. if $q > m_0$.

In order to study inflation inertia, as in the case of the evolutionary economy, let us assume the Calvo economy is initially at steady state. It is easy to see that it adjusts to an unexpected permanent shock on the growth rate of money supply by jumping to the new steady state. Remember that a change in μ only affects the steady state value of inflation, implying that the steady state value of m remains unchanged. Since at the time of the shock real balances are assumed to be at steady state, from the saddle-path properties of the model, inflation has to directly jump to the new steady state. Consequently, the Calvo model shows no inflation inertia.

At the stationary solution of the Calvo model, the economy consumes the whole endowment q , meaning that the average price is equal to the equilibrium price. At steady state, given μ and v , firms receiving a price-change signal set a price equal to $(1 + \eta) P_t$, where $\eta = \mu/v$ as in the evolutionary economy. However, this equilibrium outcome results from the forward-looking behavior of Calvo players. It is important to notice that, at the time of the shock, Calvo players automatically adjust η to its new steady state value. This is a direct implication of rational expectations. They know the economy is jumping to the new steady state and use this information to perfectly forecast present and future inflation. Importantly, they incorporate this information in their price rule, making it jump to the new steady state too. In contrast, our price pioneers are backward looking and follow a simple rule of thumb. The inflation adjustment

in the evolutionary economy takes place gradually because it relies on the Darwinian dynamics.

4 Conclusion

Calvo's (1983) method of incorporating staggered price setting in a utility maximizing framework is by now standard in the literature on New Keynesian Macroeconomics. One of the main weaknesses of the original formulation was a failure to deliver the empirically observed levels of persistence in inflation. Prominent remedies of this shortcoming include the introduction of price indexation in an otherwise standard sticky price model and the replacement of the assumption of sticky prices with sticky information. Here we have provided a simple alternative to those remedies. We have shown that, if prices are determined using an evolutionary principle, inflation inertia arises naturally even in a sticky price framework without price indexation.

In Calvo's original formulation, a fraction of firms sets prices optimally, using all available information and forming expectations rationally. This has the undesirable effect that, even if the optimizing firms are a tiny minority, their choices will be such that the overall inflation level will jump immediately to the new steady state in response to a permanent change in the money growth rate. Our evolutionary economy eventually converges to a stationary state where the average price (and inflation) is equal to the rational expectations equilibrium price (and inflation). However, in response to a change in the fundamentals, its convergence to the rational expectations inflation is only gradual. This is because no single firm realizes the changes in the economy, it is rather the economy as a whole that learns about the new environment through a process of Darwinian selection. Firms receiving a price change signal keep playing the same price rule as before. But the frequency of the price change signal (or equivalently the number of firms that receive it) evolves as it becomes more or less profitable for firms to change their price and eventually converges to a stationary level consistent with rational expectation inflation, but only after a significant period of adjustment.

A numerical example provides an indication as to the strength of inflation inertia, more detailed quantitative prediction will be presented in a follow up article.

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